

11 Sequences and Series

11-1 Types of Sequences

Objective: To determine whether a sequence is arithmetic, geometric, or neither and to supply missing terms of a sequence.

Vocabulary

Sequence A function whose domain consists of consecutive positive integers.
Each corresponding value is a *term* of a sequence.

Finite sequence A sequence that has a limited number of terms.

Infinite sequence A sequence that has an unlimited number of terms.

Arithmetic sequence (or arithmetic progression) A sequence in which the difference between any two successive terms is constant. This constant difference is called the *common difference* and is usually denoted by d .

Geometric sequence (or geometric progression) A sequence in which the ratio of every pair of successive terms is constant. This constant ratio is called the *common ratio* and is usually denoted by r .

Symbols

d (common difference) r (common ratio) t_n (n th term of a sequence)

Example 1 For each arithmetic sequence, find the common difference and the next two terms of the sequence.

- a. 5, 8, 11, 14, 17, . . .

Solution a. Find the common difference by subtracting any term from the term that follows it:
 $d = 8 - 5 = 3$

\therefore the next two terms are $17 + 3 = 20$ and $20 + 3 = 23$.

b. The common difference is $d = 16 - 23 = -7$.

\therefore the next two terms are $-5 + (-7) = -12$ and $-12 + (-7) = -19$.

Example 2 For each geometric sequence, find the common ratio and the next two terms of the sequence.

- a. 3, 6, 12, 24, . . .

b. $90, -30, 10, -\frac{10}{3}, \dots$

Solution a. Find the common ratio by dividing any term by the term before it:

$$r = \frac{6}{3} = 2$$

\therefore the next two terms are $24 \cdot 2 = 48$ and $48 \cdot 2 = 96$.

b. The common ratio is $r = \frac{-30}{90} = -\frac{1}{3}$.

\therefore the next two terms are $(-\frac{10}{3})(-\frac{1}{3}) = \frac{10}{9}$ and $(\frac{10}{9})(-\frac{1}{3}) = -\frac{10}{27}$.

11-1 Types of Sequences (continued)

Example 3 Using the given formula for the n th term, find t_1, t_2, t_3 , and t_4 . Then tell whether the sequence is arithmetic, geometric, or neither.

a. $t_n = 3 + 2n$

Substitute 1, 2, 3, and 4 for n in turn.

a. n | $t_n = 3 + 2n$

1	$t_1 = 3 + 2(1) = 5$	The sequence 5, 7, 9, 11, . . .
2	$t_2 = 3 + 2(2) = 7$	is arithmetic since the common difference is 2.
3	$t_3 = 3 + 2(3) = 9$	
4	$t_4 = 3 + 2(4) = 11$	

b. n | $t_n = \sqrt{n}$

1	$t_1 = \sqrt{1} = 1$	The sequence 1, $\sqrt{2}, \sqrt{3}, 2, \dots$
2	$t_2 = \sqrt{2}$	is neither arithmetic nor geometric.
3	$t_3 = \sqrt{3}$	
4	$t_4 = \sqrt{4} = 2$	

c. n | $t_n = -2 \cdot 3^n$

1	$t_1 = -2 \cdot 3^1 = -6$	The sequence -6, -18, -54, -162, . . .
2	$t_2 = -2 \cdot 3^2 = -18$	is geometric since the common ratio is 3.
3	$t_3 = -2 \cdot 3^3 = -54$	
4	$t_4 = -2 \cdot 3^4 = -162$	

A = arithmetic, **G** = geometric, **N** = neither 1. **A**; 25, **28** 6. **G**; 27, **243**
Tell whether each sequence is arithmetic, geometric, or neither. Then supply the missing terms of the sequence. **G**; 3, **2**

1. $13, 16, 19, 22, \underline{?}, \underline{?}$ 2. $1, 4, 16, 64, \underline{?}, \underline{?}$

4. $-3, 0, 3, 6, \underline{?}, \underline{?}$ 5. $31, 27, 23, \underline{?}, 15, \underline{?}$ 6. $-1, 3, -9, \underline{?}, -81, \underline{?}$

7. $\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{11}, \frac{6}{13}$ 8. $-2, \underline{?}, 12, 19, \frac{2}{3}, \frac{33}{26}$

Find the first four terms of the sequence with the given formula.

Then tell whether the sequence is arithmetic, geometric, or neither.

10. $t_n = 2 + 5n$ 11. $t_n = 2^{2n}$ 12. $t_n = 3^{1-n}$

14. $t_n = 3n - 1$ 15. $t_n = \frac{1}{n}$ 16. $t_n = (-3)^n$

17. $t_n = \frac{n+1}{2n}$ 18. $t_n = \frac{5}{11}, \frac{6}{13}$

Mixed Review Exercises 12. $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}; \mathbf{G}$ 15. $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}; \mathbf{N}$

Graph each equation. Graphs are given at the back of this Answer Key.

1. $y = x^2 - 4x - 1$ 2. $y = 3^{-x}$

3. $x^2 + 9y^2 = 36$ 4. $(x - 1)^2 - y^2 = 16$

5. $x + 4y = 1$ 6. $y = x^2$

3x + y = 14 7. $y = 4x^2 - 2$

(5, -1) 8. $x^2 + y^2 = 4$

(1, 1); (-2, 4) 9. $(4, -\sqrt{5}); (4, \sqrt{5})$

(-4, -\sqrt{5}); (-4, \sqrt{5})

(-4, -\sqrt{5}); (-4, \sqrt{5})

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