

11-2 Arithmetic Sequences

Objective: To find a formula for the n th term of an arithmetic sequence and to find specified terms of arithmetic sequences.

Vocabulary

n th (or general) term of an arithmetic sequence In an arithmetic sequence with first term t_1 and common difference d , the n th (or general) term is given by

$$t_n = t_1 + (n - 1)d.$$

Arithmetic means The term(s) between two given terms of an arithmetic sequence.

Example: For the arithmetic sequence 5, 9, 13, 17, ..., the numbers 9 and 13 are two arithmetic means between 5 and 17.

The arithmetic mean A single arithmetic mean between two numbers. The arithmetic mean, or *average*, of two numbers a and b is the number $\frac{a + b}{2}$.

Example: The arithmetic mean of 5 and 17 is $\frac{5 + 17}{2}$, or 11.

Example 1 Find a formula for the n th term of the arithmetic sequence 11, 17, 23, 29, ...

Solution The first term, t_1 , is 11. The common difference, d , is $17 - 11$, or 6.

Substitute 11 for t_1 and 6 for d in the formula $t_n = t_1 + (n - 1)d$.

$$\begin{aligned} t_n &= 11 + (n - 1)6 \\ &= 11 + 6n - 6 \\ &= 5 + 6n \end{aligned}$$

$$\therefore t_n = 5 + 6n$$

1. $t_n = 4 + 3n$ 2. $t_n = 5n$ 3. $t_n = 9 - 5n$ 4. $t_n = -6 + 5n$ 5. $t_n = 2 - 4n$
Find a formula for the n th term of each arithmetic sequence.

1. 7, 10, 13, 16, ...

2. 5, 10, 15, 20, ...

3. 4, -1, -6, -11, ...

4. -1, 4, 9, 14, ...

5. -2, -6, -10, -14, ...

6. 21, 29, 37, 45, ...

Example 2 Find t_{15} for the arithmetic sequence 11, 17, 23, 29, ...

Solution Use the formula $t_n = 5 + 6n$ from the solution of Example 1.

$$t_{15} = 5 + 6(15) = 95$$

Example 3 Find t_{25} for the arithmetic sequence in which $t_2 = -5$ and $t_6 = 7$.

Solution Substitute -5 for t_2 and 7 for t_6 in the formula $t_n = t_1 + (n - 1)d$ to obtain a system of equations in t_1 and d .

$$\begin{array}{rcl} t_2 = t_1 + (2 - 1)d & \longrightarrow & -5 = t_1 + d \\ t_6 = t_1 + (6 - 1)d & \longrightarrow & 7 = t_1 + 5d \end{array}$$

Solve the first equation for t_1 : $t_1 = -d - 5$.

(Solution continues on the next page.)

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Substitute $-d - 5$ for t_1 in the second equation and solve for d .

$$\begin{aligned} 7 &= -d - 5 + 5d \\ 12 &= 4d \\ d &= 3 \end{aligned}$$

Then $t_1 = -3 - 5 = -8$. Now using $t_1 = -8$ and $d = 3$ in the formula $t_n = t_1 + (n - 1)d$, find t_{25} .

$$\begin{aligned} t_{25} &= -8 + (25 - 1)(3) \\ &= -8 + 24(3) \\ &= 64 \end{aligned}$$

$$\therefore t_{25} = 64$$

Find the specified term of each arithmetic sequence.

7. 4, 7, 10, 13, ...; t_9 **58**

8. 2, 10, 18, 26, ...; t_{56} **442**

9. 90, 87, 84, 81, ...; t_{20} **33**

10. 1, 1.25, 1.5, 1.75, ...; t_{33} **9**

11. -3, -12, -21, ...; t_{50} **-444**

12. 19, 8, -3, ...; t_{41} **-421**

13. $t_1 = 2$, $t_4 = 8$; t_{14} **28**

14. $t_2 = -5$, $t_4 = -11$; t_{10} **-29**

15. $t_6 = 22$, $t_{10} = 38$; t_2 **2**

16. $t_{10} = 70$, $t_{15} = 60$; t_5 **80**

Example 4 a. Find the arithmetic mean of -3 and 8.

b. Insert three arithmetic means between 10 and 26.

Solution a. The arithmetic mean is the average of -3 and 8.

$$\frac{-3 + 8}{2} = \frac{5}{2} = 2.5$$

b. Set up the sequence: 10, $\frac{2}{2}$, $\frac{2}{2}$, $\frac{2}{2}$, 26

In this sequence, 10 is the *first* term and 26 is the *fifth* term. So to find d , substitute 10 for t_1 and 26 for t_5 in the formula $t_n = t_1 + (n - 1)d$.

$$26 = 10 + (5 - 1)d$$

$$16 = 4d$$

$$d = 4$$

The three arithmetic means are obtained by adding 4 to successive terms: 10, 14, 18, 22, 26

Find the arithmetic mean of each pair of numbers.

17. -2, 9 **3.5**

18. 3.2, 6.4 **4.8**

19. $\frac{2}{3}$, $\frac{3}{5}$ **$\frac{19}{30}$**

Insert (a) two and (b) three arithmetic means between each pair of numbers.

21. -1, 5

22. 30, 50

23. 14, 39

24. 0, 40

Additional answers for Exs. 21-24 are given at the back of this Answer Key.