NAME ______________________ DATE ______________

11–4 Series and Sigma Notation

Objective: To identify series and to use sigma notation.

1. $-1 + 1 + 3 + 5 + 7$

2. $1 + \frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \frac{1}{625}$

3. $3 + 2 + 1 + 0 + 1 + 2 + 3$

4. $-\frac{1}{7} + \frac{1}{8} - \frac{1}{9} + \frac{1}{10} - \frac{1}{11} + \frac{1}{12}$

Vocabulary

Series The indicated sum of the terms of a sequence.

Arithmetic series A series whose related sequence is arithmetic.
Example: $5 + 8 + 11 + 14$ is a *finite* arithmetic series (with four terms).

Geometric series A series whose related sequence is geometric.
Example: $3 + 6 + 12 + 24 + \cdots$ is an *infinite* geometric series.

Sigma notation The Greek letter Σ (*sigma*), called the *summation sign*, can be used to write a series in abbreviated form. Examples: The finite arithmetic series

$5 + 8 + 11 + 14$ can be written $\sum_{n=1}^{4} (3n + 2)$, which is read "the sum of $3n + 2$ for values of n from 1 to 4." The infinite geometric series $3 + 6 + 12 + 24 + \cdots$ can be written $\sum_{k=1}^{\infty} 3(2^{k-1})$, which is read "the sum of $3(2^{k-1})$ for values of k from 1 to infinity." In the first example, $3n + 2$ is called a *summand*; the letter n is called the *index*; and 1 and 4 are called the *upper* and *lower limits of summation*, respectively. Any letter can be used as the index in a summation.

Symbol

Σ (sigma; used as a summation sign) ∞ (infinity; used to denote an infinite series)

Example 1 Write the series $\sum_{j=1}^{3} (-1)^j(3j + 1)$ in expanded form.

Solution Replace j with 1, 2, and 3 in turn.

$$\sum_{j=1}^{3} (-1)^j(3j + 1) = (-1)^1(3 \cdot 1 + 1) + (-1)^2(3 \cdot 2 + 1) + (-1)^3(3 \cdot 3 + 1)$$
$$= (-1)(4) + (1)(7) + (-1)(10) = -4 + 7 - 10$$

Write each series in expanded form. See above.

1. $\sum_{n=1}^{5} (2n - 3)$ 2. $\sum_{j=0}^{4} 5^{-j}$ 3. $\sum_{k=3}^{9} |6 - k|$ 4. $\sum_{n=5}^{10} \frac{(-1)^n}{n + 2}$

Example 2 Use sigma notation to write each series.
a. $4 + 7 + 10 + \cdots + 61$ b. $3 + 6 + 12 + \cdots + 1536$

Solution a. Since the series is arithmetic with common difference 3, the nth term is:

$$t_n = t_1 + (n - 1)d = 4 + (n - 1)3 = 3n + 1$$

(*Solution continues on the next page.*)

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11–4 Series and Sigma Notation (continued)

Now find n such that the last term is 61:

$$t_n = 3n + 1$$
$$61 = 3n + 1$$
$$60 = 3n$$
$$n = 20$$

$\therefore$ the series is $\sum_{n=1}^{20} (3n + 1)$.

b. Since the series is geometric with common ratio 2, the nth term is $t_n = 3(2^{n-1})$.

Now find n such that the last term is 1536:

$$t_n = 3(2^{n-1})$$
$$1536 = 3(2^{n-1})$$
$$512 = 2^{n-1}$$
$$2^9 = 2^{n-1}$$
$$9 = n - 1 \quad \text{Equate exponents.}$$
$$n = 10$$

$\therefore$ the series is $\sum_{n=1}^{10} 3(2^{n-1})$.

Additional answers are given at the back of this Answer Key.

Write each series using sigma notation.

5. $4 + 8 + 12 + \cdots + 100$
6. $25 + 30 + 35 + \cdots + 205$
7. $2 \cdot 5 + 2 \cdot 5^2 + 2 \cdot 5^3 + \cdots + 2 \cdot 5^{15}$
8. $1 + 0.1 + 0.01 + \cdots + 0.0000001$
9. $8 + 5 + 2 + \cdots + (-40)$
10. $\frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \cdots + \frac{256}{6561}$

Example 3 Use sigma notation to write the series $\frac{2}{3} + \frac{4}{5} + \frac{6}{7} + \frac{8}{9} + \cdots$.

Solution Since the infinite series is neither arithmetic nor geometric, you need to look for patterns.

1. Since the numerators are consecutive even integers, a general expression for the numerators is $2n$.
2. Since each denominator is 1 more than the corresponding numerator, a general expression for the denominators is $2n + 1$.

So the expression for the summand is $\frac{2n}{2n + 1}$.

$\therefore$ the series is $\sum_{n=1}^{\infty} \frac{2n}{2n + 1}$.

Write each series in sigma notation.

11. $\sqrt{11} + \sqrt{22} + \sqrt{33} + \cdots + \sqrt{143}$ **11.** $\sum_{n=1}^{13} \sqrt{11n}$
12. $1^2 + 3^4 + 5^6 + 7^8 + \cdots + 99^{100}$ $\sum_{n=1}^{50} (2n-1)^{2n}$
13. $\frac{1}{3} + \frac{2}{5} + \frac{3}{7} + \frac{4}{9} + \cdots$ $\sum_{n=1}^{\infty} \frac{n}{2n + 1}$
14. $\frac{3}{2} + \frac{4}{3} + \frac{5}{4} + \frac{6}{5} + \cdots$ $\sum_{n=1}^{\infty} \frac{n + 2}{n + 1}$