

Instructions: Do all work on your own paper. SHOW YOUR WORK clearly for each problem.

Problems 1-3: Suppose $f(x) = \frac{x}{2}$, $g(x) = x - 3$, and $h(x) = \sqrt{x}$. Find a REAL number or an expression in x for each of the compositions below.

- | | | | |
|----|--------------|---------------|--------------|
| 1. | a) $f(g(8))$ | b) $f(g(-5))$ | c) $f(g(x))$ |
| 2. | a) $f(h(4))$ | b) $f(h(-4))$ | c) $f(h(x))$ |
| 3. | a) $h(g(2))$ | b) $h(g(x))$ | c) $h(h(x))$ |

Problems 4-6: Determine if the given functions are inverses. *You might want to review the definition of function inverses from your notes.*

4. $f(x) = 2x - 3$, $g(x) = \frac{x+3}{2}$
5. $f(x) = \frac{x+6}{3}$, $g(x) = 3x - 6$
6. $f(x) = \frac{1}{x+1}$, $g(x) = \frac{1}{x} + 1$

Problems 7-11: Write the domain of each function $f(x)$, then find its inverse, $f^{-1}(x)$. Write the domain of the inverse, then state whether it qualifies as the true inverse of $f(x)$.

7. $f(x) = \sqrt[3]{x}$
8. $f(x) = x^4$
9. $f(x) = -4x + 1$
10. $f(x) = \sqrt{x}$
11. $f(x) = x^2$

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Problems 1-3: Suppose $f(x) = \frac{x}{2}$, $g(x) = x - 3$, and $h(x) = \sqrt{x}$. Find a REAL number or an expression in x for each of the compositions below.

- | | | | |
|----|------------------------------|-------------------------------|---|
| 1. | a) $f(g(8))$ $\frac{5}{2}$ | b) $f(g(-5))$ -4 | c) $f(g(x))$ $\frac{x-3}{2}$ |
| 2. | a) $f(h(4))$ 1 | b) $f(h(-4))$ <i>non real</i> | c) $f(h(x))$ $\rightarrow \frac{\sqrt{x}}{2}$ |
| 3. | a) $h(g(2))$ <i>non real</i> | b) $h(g(x))$ $\sqrt{x-3}$ | c) $h(h(x))$ $\sqrt{\sqrt{x}}$ |
- $\sqrt{\sqrt{x}} = \sqrt[4]{x}$

Problems 4-6: Determine if the given functions are inverses. *You might want to review the definition of function inverses from your notes.*

4. $f(x) = 2x - 3$, $g(x) = \frac{x+3}{2}$ *yes, inverses*
5. $f(x) = \frac{x+6}{3}$, $g(x) = 3x - 6$ *yes, inverses*
6. $f(x) = \frac{1}{x+1}$, $g(x) = \frac{1}{x} + 1$ *no, not inverses*

Problems 7-11: Write the domain of each function $f(x)$, then find its inverse, $f^{-1}(x)$. Write the domain of the inverse, then state whether it qualifies as the true inverse of $f(x)$.

7. $f(x) = \sqrt[3]{x}$ *yes*
8. $f(x) = x^4$ *no*
9. $f(x) = -4x + 1$ *yes*
10. $f(x) = \sqrt{x}$ *no*
11. $f(x) = x^2$ *no*

10-3 HW Solutions

$$f(x) = \frac{x}{2} \quad g(x) = x-3 \quad h(x) = \sqrt{x}$$

① a) $f(g(8)) = \frac{5}{2}$ b) $f(g(-5)) = -4$ c) $f(g(x)) = \frac{x-3}{2}$
 $g(8) = 8-3 = 5$ $g(-5) = -5-3 = -8$ $f(x-3) = \frac{x-3}{2}$
 $f(5) = \frac{5}{2}$ $f(-8) = \frac{-8}{2} = -4$

② $f(h(4)) = 1$ $f(h(-4))$ $f(h(x)) = \frac{\sqrt{x}}{2}$
 $h(4) = \sqrt{4} = 2$ $h(-4) = \sqrt{-4}$ "non real!" $f(\sqrt{x}) = \frac{\sqrt{x}}{2}$
 $f(2) = \frac{2}{2} = 1$

③ $h(g(2)) =$ $h(g(x)) = \sqrt{x-3}$ $h(h(x)) = \sqrt[4]{x}$
 $g(2) = 2-3 = -1$ $h(x-3) = \sqrt{x-3}$ $h(\sqrt{x}) = \sqrt{\sqrt{x}} = \sqrt[4]{x}$
 $h(-1) = \sqrt{-1}$ "non real!"

④ $f(x) = 2x-3$ $g(x) = \frac{x+3}{2}$ ⑤ $f(x) = \frac{x+6}{3}$ $g(x) = 3x-6$
 $y = 2x-3$ $y = \frac{x+3}{2}$ $y = \frac{x+6}{3}$ $y = 3x-6$
 $x = 2y-3$ $x = \frac{y+3}{2}$ $x = \frac{y+6}{3}$ $x = 3y-6$
 $\frac{x+3}{2} = \frac{2y}{2}$ $2x = y+3$ $3x = y+6$ $\frac{x+6}{3} = \frac{3y}{3}$
 $f^{-1}(x) = \frac{x+3}{2}$ $2x-3 = y$ $3x-6 = y$ $y = \frac{x+6}{3}$
 $f^{-1}(x) = 2x-3$ $f^{-1}(x) = 2x-3$ $f^{-1}(x) = 3x-6$ $f^{-1}(x) = \frac{x+6}{3}$

INVERSES! INVERSES!

$$\textcircled{6} \quad f(x) = \frac{1}{x+1} \quad g(x) = \frac{1}{x} + 1$$

$$y = \frac{1}{x+1}$$

$$x = \frac{1}{y+1}$$

$$\frac{x(y+1)}{x} = \frac{1}{x}$$

~~$$y+1 = \frac{1}{x}$$~~

$$y+1 = \frac{1}{x}$$

$$-1 \quad -1$$

$$y = \frac{1}{x} - 1$$

$$\underline{\underline{f^{-1}(x) = \frac{1}{x} - 1}}$$

does not match!

$$y = \frac{1}{x} + 1$$

$$x = \frac{1}{y} + 1$$

$$x-1 = \frac{1}{y}$$

$$y \frac{(x-1)}{x-1} = \frac{1}{x-1}$$

$$\underline{\underline{y = \frac{1}{x-1}}}$$

does not match!

$$\textcircled{7} \quad \text{a) } \{ \text{all reals} \}$$

$$\text{b) } x^3 = f^{-1}(x)$$

$$\text{c) } \{ \text{all reals} \}$$

d) true inverses

$$\textcircled{8} \quad \text{a) } \{ \text{all reals} \}$$

$$\text{b) } \sqrt[4]{x} = f^{-1}(x)$$

$$\text{c) } \{ x : x \geq 0 \}$$

d) NOT true inverses

$$\textcircled{9} \quad \text{a) } \{ \text{all reals} \}$$

$$\text{b) } y = -4x + 1$$

$$x = -4y + 1$$

$$\frac{x-1}{-4} = \frac{-4y}{-4}$$

$$f^{-1}(x) = \frac{x-1}{-4}$$

$$\text{c) } \{ \text{all reals} \}$$

d) true inverses

$$\textcircled{10} \quad f(x) = \sqrt{x}$$

$$\text{a) } \{ x : x \geq 0 \}$$

$$\text{b) } f^{-1}(x) = x^2$$

$$\text{c) } \{ \text{all reals} \}$$

d) NOT true inverses

$$\textcircled{11} \quad f(x) = x^2$$

$$\text{a) } \{ \text{all reals} \}$$

$$\text{b) } f^{-1}(x) = \sqrt{x}$$

$$\text{c) } \{ x : x \geq 0 \}$$

d) NOT true inverses