Instructions: Do all work on your own paper. Show all steps that lead to your solution. If a problem requires graphing, use graph paper and a straightedge for lines.

Section 3-8

- Definitions:
  - **Function**
  - **Domain**
  - **Range**

use notes or text

- Graph a function over a given domain.
- Evaluate a function at a given input, for example, find f(2) if f(x) = x + 3
- Evaluate composite functions, for example find f(g(3)) if f(x) = x + 3 and g(x) = 2x – 5.
- Determine the range of a function over a given domain.
- Determine the domain of a function by finding the places where it "blows up" (becomes undefined).

1. Given that $f(x) = -3x^2 + 10$ and $g(x) = |x-4| - 6$, find the following:

(A) $\boxed{f(-2) = -2}$ (B) $\boxed{g(f(3)) = 15}$ (C) $\boxed{f(g(-2)) = 10}$

$f(-2) = -3(-2)^2 + 10$
$-3(4) + 10$
$-12 + 10$
$f(-2) = -2$

$f(3) = -3(3)^2 + 10$
$= -27 + 10$
$= -17$
$g(-17) = |-17-4| - 6$
$|-21| - 6 = 21 - 6 = \boxed{15}$

$g(-2) = |-2-4| - 6$
$|-6| - 6$
$6 - 6$
0
$f(0) = -3(0)^2 + 10 = 10$

2. Find the domain of each function.

(A) $f(x) = \sqrt{3x-2}$ (B) $g(x) = \dfrac{2}{(x-1)(x+4)}$ (C) $h(x) = \dfrac{\sqrt{3-x}}{x^2-25}$

$3x - 2 = 0$
$x = 2/3$
$\boxed{\{x : x \geq \tfrac{2}{3}\}}$
0 $\tfrac{2}{3}$ 1

$x - 1 = 0$
$x = 1$
$x + 4 = 0$
$x = -4$
All reals except 1 & −4
OR {all reals, $x \neq 1, -4$}

$x \leq 3$
$x \neq -5, 5$
$\boxed{\{x : x \leq 3,\ x \neq 5, -5\}}$

3. Find the range of $F(x) = |1 - x|$ over the domain D = {-2, -1, 0, 1, 2}

x	y
−2	
−1	
0	
1	
2	

$|1-(-2)| = |3|$ 3
$|1-(-1)| = |2|$ 2
$|1-(0)| = |1|$ 1
$|1-(1)| = |0|$ 0
$|1-(2)| = |-1|$ 1

$\boxed{R = \{0, 1, 2, 3\}}$

* no duplicates in answer

Section 3-9

- Definitions:
  - **linear function**
  - **constant function**
  - **rate of change**
- Determine the equation of a linear function based on given information, for example either the slope and a point or two points.
- Evaluate functions after finding the equation of the function.
- Solve word problems by creating a linear function and evaluating it according to the problem.

1. Find an equation of the linear function f for the given information and find $f(-10)$ for each.

(A) $f(0) = 3$ and $f(x)$ decreases by 2 when x increases by 3.

$b = 3$ $\quad \frac{\Delta f(x)}{\Delta x} = \frac{-2}{3}$ $\quad m = -\frac{2}{3}$

$\boxed{f(x) = -\frac{2}{3}x + 3}$ $\quad \boxed{f(-10) = \frac{29}{3}}$

(B) $f(4) = -5$ and $f(-3) = 3$

$(4, -5) \quad (-3, 3)$

$m = \frac{-5-3}{4-(-3)} = \frac{-8}{7}$

$f(x) = -\frac{8}{7}x + b$

$-5 = -\frac{8}{7}\left(\frac{4}{1}\right) + b$

$-5 = -\frac{32}{7} + b$

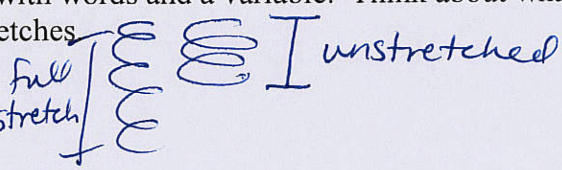
$-\frac{35}{7} + \frac{32}{7} = b$

$-\frac{3}{7} = b$

$\boxed{f(x) = -\frac{8}{7}x - \frac{3}{7} \quad f(-10) = -\frac{59}{7}}$

2. When a mass of 8 kg is attached to a spring, the spring stretches to a length of 76 cm. A mass of 14 kg stretches the same spring to a length of 85 cm.

(A) Draw a function box and label the input and output with words and a variable. Think about what is happening...when you hang a weight on a spring, it stretches
(B) Find a linear function using the given information.
(C) Find the natural (***un-stretched***) length of the spring.



A) $\text{mass} \xrightarrow{x} \boxed{f(x)} \xrightarrow{f(x)} \text{length}$

B) f(mass) = length

$f(8) = 76 \quad f(14) = 85$

$(8, 76) \quad (14, 85)$

$m = \frac{76-85}{8-14} = \frac{-9}{-6} = \frac{3}{2}$

$f(x) = \frac{3}{2}x + b$

$76 = \frac{3}{2}(8) + b$

$76 = 12 + b$

$-12 \quad -12$

$64 = b$

$\boxed{f(x) = \frac{3}{2}x + 64}$

C) $f(0) = 64$

0 mass would show the unstreched length

$\boxed{\text{The unstretched spring is 64 cm.}}$

3. Suppose the number of hours an algebra student studies and the grade on a test is a linear function. A student who spent 1 hour studying earned a 76. A student who spent 3 hours earned an 88.

(A) Find a linear function using the given information. Think about the input and output first.

hours $\rightarrow \boxed{f(x)} \rightarrow$ grade

$f(\text{hours}) = \text{grade}$

$f(1) = 76$

$f(3) = 88$

$m = \dfrac{88-76}{3-1} = \dfrac{12}{2} = 6$

$f(x) = 6x + b$

$76 = 6(1) + b$

$-6 \qquad -6$

$70 = b$

$\boxed{f(x) = 6x - 70}$

(B) What score would a student who studied for 4 hours earn?

Input

$f(4) = 6(4) + 70$

$f(4) = 24 + 70$

$= 94\%$

They would earn 94%.

(C) How many hours would a student need to study to earn a grade of 80?

output

$80 = 6x + 70$

$-70 \qquad -70$

$10 = 6x$

$\dfrac{10}{6} = \dfrac{6x}{6}$

$x = \dfrac{5}{3} = 1\dfrac{2}{3} =$ 1 hr 40 min

They would need to study 1 hour + 40 minutes to earn an 80%.

Advanced Algebra **Review Guide 3-10, 10-3** **Mrs. Dunphy**

Name ____________________

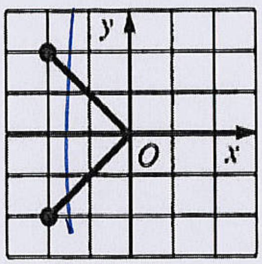
Section 3-10

Definitions: relation and function (using the definition of relation), vertical lines test

- Determine if a relation is a function by inspecting the ordered pairs or using the vertical line test on a graph of the function.
- Find the domain of a relation, graph it, and determine if it is a function.

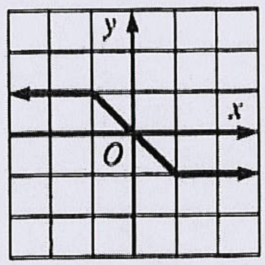
2-4: Give the domain and range of each function shown. Then state whether it is a function or not.

2.



NOT a function (fails VLT)

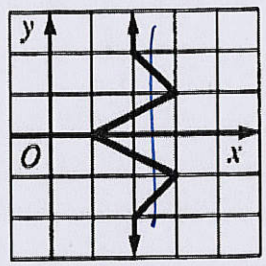
3.



D = {all reals}

R = {all reals -1 to 1}

4.



NOT a function (fails VLT)

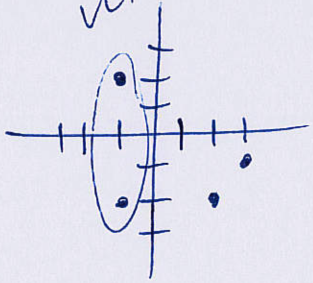
9-10: Determine if each relation is a function. If not, state why.

9. $\{(-1, 2), (3, -1), (2, -2), (-1, -2)\}$

x -1 is a duplicate x value

NOT a function

fails VLT!

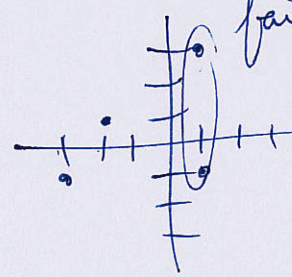


10. $\{(1, 3), (3, -1), (1, -1), (-2, -1)\}$

1 is a duplicate x value

NOT a function

fails VLT



Section 10-3

Definitions: composite, composition, inverse function

- Find the ***composite*** of two functions: **h(x) = f(g(x))**.
- State the meaning on inverse functions: Two functions are inverses only if f(g(x)) = x and g(f(x)) = x.
- Simple example: $f(x) = x^2$ and $g(x) = \sqrt{x}$ because $f(g(x)) = (\sqrt{x})^2 = x$ *and* $g(f(x)) = \sqrt{x^2} = x$.
- Find the inverse of a function, if it exists: (1) write y = f(x); (2) swap x and y; (3) solve for y; (4) replace y with $f^{-1}(x)$; (5) check the domains

1-2: Suppose $f(x) = \frac{x}{2}$, $g(x) = x - 3$, *and* $h(x) = \sqrt{x}$. Find a real-number value or an expression in x for each of the following. If no real value can be found, say so.

1. a. $\boxed{h(f(72)) = 6}$
$f(72) = 36$
$h(36) = \sqrt{36}$
6

b. $\boxed{h(f(50)) = 5}$
$f(50) = 25$
$h(25) = \sqrt{25} = 5$

c. $\boxed{h(f(x)) = \sqrt{\frac{x}{2}}}$
$h\left(\frac{x}{2}\right) = \sqrt{\frac{x}{2}}$

d. $\boxed{f(f(x)) = \frac{x}{4}}$
$f\left(\frac{x}{2}\right) = \frac{\frac{x}{2}}{2}$
$\frac{x}{2} \cdot \frac{1}{2} = \frac{x}{4}$

2. a. $\boxed{g(h(9)) = 0}$
$h(9) = 3$
$g(3) = 3 - 3 = 0$

b. $\boxed{g(h(3)) = \sqrt{3} - 3}$
$h(3) = \sqrt{3}$
$g(\sqrt{3}) = \sqrt{3} - 3$

c. $\boxed{g(h(x)) = \sqrt{x} - 3}$
$g(\sqrt{x}) = \sqrt{x} - 3$

d. $\boxed{g(g(x)) = x - 6}$
$g(x-3) = (x-3) - 3$
$= x - 6$

3-5: Find the inverse of each function algebraically.

3. $f(x) = 2x - 3$
$y = 2x - 3$
$x = 2y - 3$
$x + 3 = 2y$
$\frac{x+3}{2} = y$
$\boxed{f^{-1}(x) = \frac{x+3}{2}}$

4. $g(x) = \frac{1}{x-1}$
$y = \frac{1}{x-1}$
$x = \frac{1}{y-1}$
$\frac{x(y-1)}{x} = \frac{1}{x}$
$y - 1 = \frac{1}{x}$
$+1 \quad +1$
$y = \frac{1}{x} + 1$
$\boxed{f^{-1}(x) = \frac{1}{x} + 1}$

5. $h(x) = 2x^3 - 1$
$y = 2x^3 - 1$
$x = 2y^3 - 1$
$\frac{x+1}{2} = \frac{2y^3}{2}$
$\sqrt[3]{\frac{x+1}{2}} = \sqrt[3]{y^3}$
$\boxed{\sqrt[3]{\frac{x+1}{2}} = f^{-1}(x)}$