Answers

Advanced Algebra **Review Guide 3-8, 3-9, 3-10, 10-3** **Mrs. Dunphy**

Instructions: Show all steps that lead to your solution. Check your answer on the blog.

Section 3-8

Definitions:

- **Function**
- **Domain**
- **Range**

SKILLS TO KNOW:

- Graph a function over a given domain.
- Evaluate a function at a given input, for example, find f(2) if f(x) = x + 3
- Evaluate composite functions, for example find f(g(3)) if f(x) = x + 3 and g(x) = 2x – 5.
- Determine the range of a function over a given domain.
- Determine the domain of a function by finding the places where it "blows up" (becomes undefined).

1. Given that $f(x) = -3x^2 + 10$ and $g(x) = |x - 4| - 6$, find the following:

(A) $f(-2) = -2$ (B) $f(4) = -38$ (C) $f(0) = 10$

(D) $g(-6) = 4$ (E) $g(4) = -6$ (F) $g(0) = -2$

(G) $g(f(3)) = 15$ (H) $f(g(-2)) = 10$ (I) $g(g(g(-2))) = 0$

(J) $(f+g)(2) = -6$ (K) $(f-g)(3) = -12$ (L) $(f \cdot g)(-2) = 0$

(M) $(f/g)(4) = \frac{19}{3}$ (N) $f(2g(-3)) = -2$

2. Find the domain of each function.

(A) $f(x)=\sqrt{3x-2}$ (B) $g(x)=\dfrac{2}{(x-1)(x+4)}$ (C) $h(x)=\dfrac{\sqrt{3-x}}{x^2-25}$

$\{x: x \geq \frac{2}{3}\}$ $\{x: x \neq 1, -4\}$ $\{x: x \leq 3, x \neq -5\}$

(you do not need to state that $x \neq +5$)

3. Find the range of $F(x)=|1-x|$ over the domain D = {-2, -1, 0, 1, 2}

x	f(x)
-2	3
-1	2
0	1
1	0
2	1

$\{0,1,2,3\}$

(no repeats)

Section 3-9

Definitions:

- **linear function**
- **constant function,**
- **rate of change**

SKILLS TO KNOW:

- Determine the equation of a linear function based on given information, for example either the slope and a point or two points.
- Evaluate functions after finding the equation of the function.
- Solve word problems by creating a linear function and evaluating it according to the problem.

1. Find an equation of the linear function f for the given information and find $f(-10)$ for each.

(A) $f(0)=3$ and $f(x)$ decreases by 2 when x increases by 3.

$b=3$ or $(0,3)$ $m=\frac{-2}{3}$

$f(x)=-\frac{2}{3}x+3$

$f(-10)=9\frac{2}{3}$ or $\frac{29}{3}$

(B) $f(4)=-5$ and $f(-3)=3$

$(4,-5)$ $(-3,3)$

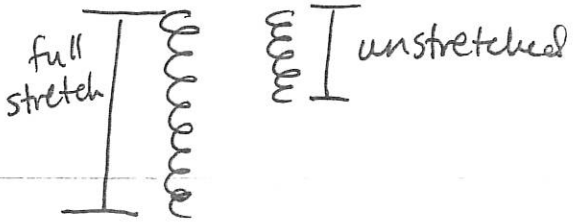
$f(x)=-\frac{8}{7}x-\frac{3}{7}$

$f(-10)=11$

2. When a mass of 8 kg is attached to a spring, the spring stretches to a length of 76 cm. A mass of 14 kg stretches the same spring to a length of 85 cm.

(A) Draw a function box and label the input and output with words and a variable.
Think about what is happening...when you hang a weight on a spring, it stretches.

mass (x) → [f(x)] → length f(x)



(B) Find a linear function using the given information.

$f(\text{mass}) = \text{length}$

$f(x) = \frac{3}{2}x + 64$

(C) Find the natural (***un-stretched***) length of the spring

$f(0) = 64$ The unstretched spring is 64 cm long.

3. Suppose the number of hours an algebra student studies and the grade on a test is a linear function. A student who spent 1 hour studying earned a 76. A student who spent 3 hours earned an 88.

(A) Find a linear function using the given information. Think about the input and output first.

$f(\text{hours}) = \text{grade}$ $f(1)=76$ $f(3)=88$

$f(x) = 6x - 70$

(B) What score would a student who studied for 4 hours earn?
(4 hours: input)

$f(4) = 94$ That student would earn a 94%.

(C) How many hours would a student need to study to earn a grade of 80?
(80: output)

$80 = 6x + 70$

$x = \frac{5}{3} = 1\frac{2}{3} = 1 \text{ hr}, 40 \text{ min}$

They would need to study for 1 hour and 40 minutes to earn an 88%.

Section 3-10

Definitions:

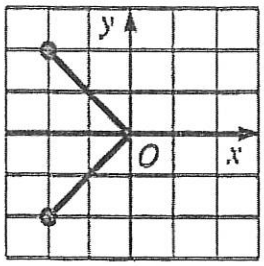
- **relation**
- **function**
- **vertical lines test**

SKILLS TO KNOW:

- Determine if a relation is a function by inspecting the ordered pairs or using the vertical line test on a graph of the function.
- Find the domain of a relation, graph it, and determine if it is a function.

2-4: Give the domain and range of each function shown. Then state whether it is a function or not.

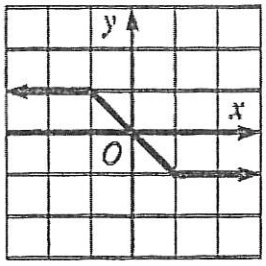
2.



$D = \{x: -2 \le x \le 0\}$

$R = \{y: -2 \le y \le 2\}$

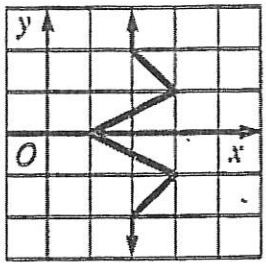
3.



$D = \{\text{all reals}\}$

$R = \{y: -4 \le y \le 4\}$

4.

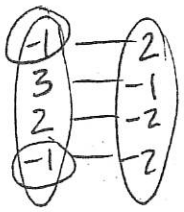


$D = \{x: -1 \le x \le 3\}$

$R = \{\text{all reals}\}$

9-10: Determine if each relation is a function. If not, state why.

9. $\{(-1, 2), (3, -1), (2, -2), (-1, -2)\}$



$-1 \to 2$
$\to -2$

Repeated x value!

NOT a function!

10. $\{(1, 3), (3, -1), (1, -1), (-2, -1)\}$

$\to 3$
$\to -1$

Repeated x value

NOT a function!

Section 10-3

Definitions:
Composite
Composition
Inverse function

SKILLS TO KNOW:

- Find the ***composite*** of two functions: **h(x) = f(g(x))**.
- State the meaning on inverse functions: Two functions are inverses only if f(g(x)) = x and g(f(x)) = x.
- Simple example: $f(x) = x^2$ and $g(x) = \sqrt{x}$ because $f(g(x)) = \left(\sqrt{x}\right)^2 = x$ *and* $g(f(x)) = \sqrt{x^2} = x$.
- Find the inverse of a function, if it exists: (1) write y = f(x); (2) swap x and y; (3) solve for y; (4) replace y with $f^{-1}(x)$; (5) check the domains

1-2: Suppose $f(x) = \frac{x}{2}$, $g(x) = x - 3$, *and* $h(x) = \sqrt{x}$. Find a real-number value or an expression in x for each of the following. If no real value can be found, say so.

1. a. $h(f(72)) = 6$ b. $h(f(50)) = 5$ c. $h(f(x)) = \sqrt{\frac{x}{2}}$ d. $f(f(x)) = \frac{x}{4}$

2. a. $g(h(9)) = 0$ b. $g(h(3)) = \sqrt{3} - 3$ c. $g(h(x)) = \sqrt{x} - 3$ d. $g(g(x)) = x - 6$

3-5: Find the inverse of each function algebraically.

3. $f(x) = 2x - 3$ $f^{-1}(x) = \frac{x+3}{2}$

4. $g(x) = \frac{1}{x-1}$ $f^{-1}(x) = \frac{1}{x} + 1$

5. $h(x) = 2x^3 - 1$ $f^{-1}(x) = \sqrt[3]{\frac{x+1}{2}}$