

5-6 Sums and Differences of Rational Expressions

Objective: To add and subtract rational expressions.

** Read the examples on this worksheet and complete problems #1-20 All on a separate sheet of paper.*

Vocabulary

Least common multiple (LCM) of two or more polynomials The common multiple having least degree and least positive factors.

Examples: The LCM of $2x$, $6x^2$, and $3x$ is $6x^2$.

The LCM of $x(x+1)$ and $(x+1)(x-1)$ is $x(x+1)(x-1)$.

The LCM of $x+2$ and $x+4$ is $(x+2)(x+4)$.

Example 1 Simplify: a. $\frac{5}{6} + \frac{13}{6} - \frac{7}{6}$ b. $\frac{2x-5}{x-3} - \frac{x-1}{x-3}$

Solution With fractions having the same denominator, add or subtract the numerators and write the result over the common denominator.

$$\begin{aligned} \text{a. } \frac{5}{6} + \frac{13}{6} - \frac{7}{6} &= \frac{5+13-7}{6} & \text{b. } \frac{2x-5}{x-3} - \frac{x-1}{x-3} &= \frac{2x-5-(x-1)}{x-3} \\ &= \frac{11}{6} & &= \frac{2x-5-x+1}{x-3} \\ & & &= \frac{x-4}{x-3} \end{aligned}$$

Example 2 Simplify: a. $\frac{5}{6} + \frac{1}{8} - \frac{1}{3}$ b. $\frac{1}{3a} - \frac{1}{4a} + \frac{2}{a^2}$

Solution With fractions having different denominators, rewrite the fractions using their *least common denominator (LCD)*, which is the LCM of the denominators.

a. $6 = 2 \cdot 3$ and $8 = 2^3$. So the LCD = $2^3 \cdot 3 = 24$.

$$\begin{aligned} \frac{5}{6} + \frac{1}{8} - \frac{1}{3} &= \frac{5 \cdot 4}{6 \cdot 4} + \frac{1 \cdot 3}{8 \cdot 3} - \frac{1 \cdot 8}{3 \cdot 8} \\ &= \frac{20}{24} + \frac{3}{24} - \frac{8}{24} \\ &= \frac{20+3-8}{24} = \frac{15}{24} = \frac{5}{8} \end{aligned}$$

b. The LCD for $3a$, $4a$, and a^2 is $12a^2$.

$$\begin{aligned} \frac{1}{3a} - \frac{1}{4a} + \frac{2}{a^2} &= \frac{1 \cdot 4a}{3a \cdot 4a} - \frac{1 \cdot 3a}{4a \cdot 3a} + \frac{2 \cdot 12}{a^2 \cdot 12} \\ &= \frac{4a}{12a^2} - \frac{3a}{12a^2} + \frac{24}{12a^2} \\ &= \frac{4a-3a+24}{12a^2} \\ &= \frac{a+24}{12a^2} \end{aligned}$$

#1-20
All

5-6 Sums and Differences of Rational Expressions (continued)

Simplify.

1. $\frac{7}{8} - \frac{3}{8} + \frac{1}{8}$

2. $\frac{1}{2} + \frac{1}{3} + \frac{3}{5}$

3. $\frac{5}{6} + \frac{2}{5} - \frac{8}{15}$

4. $\frac{3}{4} + \frac{5}{18} - \frac{7}{9}$

5. $\frac{5}{2x} - \frac{3}{2x}$

6. $\frac{3}{5x^3y} - \frac{2}{xy^2}$

7. $\frac{x}{x+1} + \frac{1}{x+1}$

8. $\frac{8t+4}{t-2} - \frac{6t-1}{t-2}$

9. $\frac{2}{3z} + \frac{7}{12z}$

10. $\frac{3}{rs} - \frac{4}{rs^2}$

11. $\frac{3m-2}{6} - \frac{m-3}{9}$

12. $\frac{2n+1}{3n} + \frac{2-3n}{4n}$

Example 3 Simplify $\frac{3}{x^2+x-2} - \frac{5}{x^2-x-6}$.**Solution**

$$x^2 + x - 2 = (x + 2)(x - 1)$$

$$x^2 - x - 6 = (x - 3)(x + 2)$$

{	Factor the denominators
	to find the LCD.

So the LCD is $(x + 2)(x - 1)(x - 3)$.

$$\begin{aligned} \frac{3}{x^2+x-2} - \frac{5}{x^2-x-6} &= \frac{3}{(x+2)(x-1)} - \frac{5}{(x-3)(x+2)} \\ &= \frac{3(x-3)}{(x+2)(x-1)(x-3)} - \frac{5(x-1)}{(x-3)(x+2)(x-1)} \\ &= \frac{3(x-3) - 5(x-1)}{(x+2)(x-1)(x-3)} \\ &= \frac{3x-9-5x+5}{(x+2)(x-1)(x-3)} \\ &= \frac{-2x-4}{(x+2)(x-1)(x-3)} \\ &= \frac{-2(x+2)}{\cancel{(x+2)}(x-1)(x-3)} \\ &= \frac{-2}{(x-1)(x-3)}, \text{ or } -\frac{2}{(x-1)(x-3)} \end{aligned}$$

Simplify.

13. $\frac{2}{k-3} + \frac{4}{k+3}$

14. $\frac{c+1}{c} - \frac{c}{c+1}$

15. $\frac{y}{y-1} + \frac{4}{y+1}$

16. $\frac{5m+1}{2m^2-2m} - \frac{3}{2m-2}$

17. $\frac{1}{x^2-3x} - \frac{1}{x^2-9}$

18. $\frac{1}{z^2-4} + \frac{1}{(z-2)^2}$

19. $\frac{3}{p^2-3p+2} - \frac{2}{p^2-1}$

20. $\frac{1}{x^2+x-2} + \frac{1}{x^2-5x+4}$