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## 6–7 The Imaginary Number *i*

**Objective:** To use the number $i$ to simplify square roots of negative numbers.

### Vocabulary

**The number *i*** The basic element of the set of *imaginary numbers*. It is defined as follows: $i = \sqrt{-1}$, and $i^2 = -1$. The number $i$ is used to simplify square roots of negative numbers. For instance, if $r$ is a positive real number, then $\sqrt{-r} = i\sqrt{r}$.
Examples: $\sqrt{-3} = i\sqrt{3}$ $\quad \sqrt{-36} = i\sqrt{36} = 6i$

**Pure imaginary number** Any number of the form $bi$, $b \neq 0$. Examples: $3i$ and $i\sqrt{7}$

**CAUTION** When $a$ and $b$ are negative, $\sqrt{a} \cdot \sqrt{b} \neq \sqrt{ab}$.

For example: $\sqrt{-4} \cdot \sqrt{-9} \neq \sqrt{36}$

Correct: $\sqrt{-4} \cdot \sqrt{-9} = 2i \cdot 3i = 6i^2 = 6(-1) = -6$

To avoid making mistakes, always express the square root of a negative number as a pure imaginary number before performing any other operation.

***Example 1*** Simplify: **a.** $\sqrt{-98}$ **b.** $\sqrt{-9} \cdot \sqrt{-25}$

***Solution***

**a.** $\sqrt{-98} = i\sqrt{98} = i\sqrt{49 \cdot 2} = 7i\sqrt{2}$

**b.** $\sqrt{-9} \cdot \sqrt{-25} = i\sqrt{9} \cdot i\sqrt{25} = 3i \cdot 5i = 15i^2 = -15$

**Simplify.**

1. $\sqrt{-49}$ **$7i$**
2. $\sqrt{-10}$ **$i\sqrt{10}$**
3. $-3\sqrt{-144}$ **$-36i$**
4. $\sqrt{-28}$ **$2i\sqrt{7}$**
5. $3\sqrt{-12}$ **$6i\sqrt{3}$**
6. $7i \cdot 5i$ **$-35$**
7. $\sqrt{7} \cdot \sqrt{-14}$ **$7i\sqrt{2}$**
8. $\sqrt{-6} \cdot \sqrt{-15}$ **$-3\sqrt{10}$**
9. $(6i)^2$ **$-36$**
10. $(-3i)^2$ **$-9$**
11. $(-i\sqrt{5})^2$ **$-5$**
12. $(2i\sqrt{3})^2$ **$-12$**

***Example 2*** Simplify: **a.** $\frac{4}{5i}$ **b.** $\frac{10}{\sqrt{-5}}$

***Solution*** To rationalize the denominator of a fraction, you must eliminate the imaginary number $i$ from the denominator. Use the fact that $i^2 = -1$.

**a.** $\frac{4}{5i} = \frac{4}{5i} \cdot \frac{i}{i} = \frac{4i}{5i^2} = \frac{4i}{5(-1)} = -\frac{4i}{5}$

**b.** $\frac{10}{\sqrt{-5}} = \frac{10}{i\sqrt{5}} = \frac{10}{i\sqrt{5}} \cdot \frac{i\sqrt{5}}{i\sqrt{5}} = \frac{10i\sqrt{5}}{\sqrt{25}i^2} = \frac{10i\sqrt{5}}{5(-1)} = -2i\sqrt{5}$







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## 6–7 The Imaginary Number *i* *(continued)*

**Simplify.**

13. $\frac{5}{i}$ **$-5i$**
14. $\frac{6}{7i}$ **$-\frac{6i}{7}$**
15. $\frac{1}{\sqrt{-3}}$ **$-\frac{i\sqrt{3}}{3}$**
16. $-\frac{9}{\sqrt{-9}}$ **$3i$**
17. $\frac{\sqrt{24}}{3i\sqrt{8}}$ **$-\frac{i\sqrt{3}}{3}$**
18. $\frac{\sqrt{48}}{2i\sqrt{3}}$ **$-2i$**
19. $\frac{\sqrt{56}}{\sqrt{-7}}$ **$-2i\sqrt{2}$**
20. $-\frac{\sqrt{21}}{\sqrt{-35}}$ **$\frac{i\sqrt{15}}{5}$**

***Example 3*** Solve $3x^2 + 23 = 5$.

***Solution***

$3x^2 + 23 = 5$
$3x^2 = -18$
$x^2 = -6$
$x = \pm\sqrt{-6}$
$x = \pm i\sqrt{6}$

$\therefore$ the solution set is $\{i\sqrt{6}, -i\sqrt{6}\}$.

**Solve.**

21. $x^2 + 100 = 0$ **$\{\pm 10i\}$**
22. $y^2 + 81 = 0$ **$\{\pm 9i\}$**
23. $2z^2 = -128$ **$\{\pm 8i\}$**
24. $7a^2 = -28$ **$\{\pm 2i\}$**
25. $3b^2 + 28 = 4$ **$\{\pm 2i\sqrt{2}\}$**
26. $4x^2 + 78 = 6$ **$\{\pm 3i\sqrt{2}\}$**

***Example 4*** Simplify: **a.** $\sqrt{-20} + \sqrt{-45}$ **b.** $\sqrt{-20} \cdot \sqrt{-45}$

***Solution***

**a.** $\sqrt{-20} + \sqrt{-45} = i\sqrt{20} + i\sqrt{45} = 2i\sqrt{5} + 3i\sqrt{5} = (2 + 3)i\sqrt{5} = 5i\sqrt{5}$

**b.** $\sqrt{-20} \cdot \sqrt{-45} = i\sqrt{20} \cdot i\sqrt{45} = 2i\sqrt{5} \cdot 3i\sqrt{5} = [2 \cdot 3 \cdot (\sqrt{5})^2]i^2 = 30i^2 = -30$

**Simplify.**

27. a. $\sqrt{-5} + \sqrt{-80}$ **a. $5i\sqrt{5}$** b. $\sqrt{-5} \cdot \sqrt{-80}$ **b. $-20$**
28. a. $4\sqrt{-3} - \sqrt{-75}$ **a. $-i\sqrt{3}$** b. $4\sqrt{-3} \cdot (-\sqrt{-75})$ **b. 60**
29. a. $i\sqrt{27} + \sqrt{-12}$ **a. $5i\sqrt{3}$** b. $i\sqrt{27} \cdot \sqrt{-12}$ **b. $-18$**
30. a. $2i\sqrt{32} - \sqrt{-72}$ **a. $2i\sqrt{2}$** b. $2i\sqrt{32} \cdot (-\sqrt{-72})$ **b. 96**

### Mixed Review Exercises

**Solve. If an equation has no real solution, say so.**

1. $\sqrt{3x + 1} = 5$ **$\{8\}$**
2. $n^2 = 9n - 20$ **$\{4, 5\}$**
3. $\sqrt{z^2 + 75} = 2z$ **$\{5\}$**
4. $\frac{4y + 5}{7} = \frac{y - 1}{2}$ **$\{-17\}$**
5. $\frac{1}{x} + \frac{3}{x - 3} = \frac{5}{x(x - 3)}$ **$\{2\}$**
6. $\sqrt[3]{t} + 12 = 7$ **$\{-125\}$**
7. $x = \sqrt{x + 12}$ **$\{4\}$**
8. $2|m| - 5 = 1$ **$\{3, -3\}$**
9. $w = 3 + \sqrt{w + 3}$ **$\{6\}$**

**Classify each real number as either rational or irrational.**

10. $\sqrt[3]{-8}$ **rational**
11. $5.7\overline{62}$ **rational**
12. $\sqrt{24}$ **irrational**
13. 0.10111213... **irrational**