

Name: Kay

Date: _____

Instructions: You may use your calculator for all problems, though you must show the appropriate algebraic steps as necessary to communicate your thinking. Read each problem carefully!

1. Convert the quadratic function $f(x) = 3x^2 - 12x - 8$ to standard vertex form. (5 pts)

$$\begin{aligned} 8 &= 3x^2 - 12x \\ 8 + \frac{3 \cdot 4}{12} &= 3(x^2 - 4x + \underline{\frac{4}{12}}) \\ 20 &= 3(x-2)^2 \\ y &= 3(x-2)^2 - 20 \end{aligned}$$

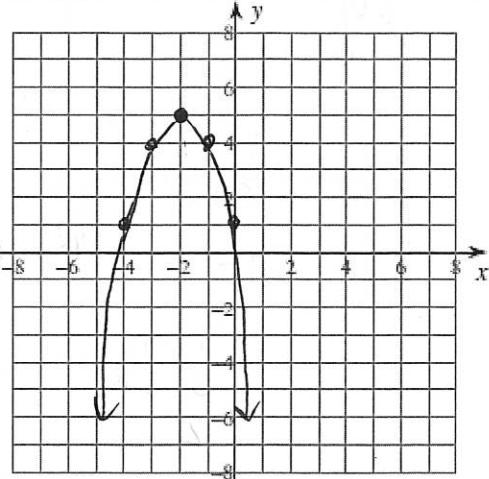
OR ~~$3(3x^2 - 12x - 8 = 0)$~~

$$\begin{aligned} 9x^2 - 36x - 24 &= 0 \\ +24 &+24 \\ 9x^2 - 36x + \underline{36} &= 24 + \underline{36} \\ 3^2(x-2)^2 &= 60 \\ a \uparrow \text{does not match!! (needs to be 3)} & \\ \rightarrow \text{divide by 3} & \\ 3(x-2)^2 &= 20 \\ 3(x-2)^2 - 20 &= y \end{aligned}$$

2. Convert the quadratic function

$g(x) = -x^2 - 4x + 1$ to standard vertex form and specify the vertex, axis of symmetry, value of the maximum or minimum, and find its zeros. algebraically. Graph the function and identify all of the above on the graph. (10 pts total)

$$\begin{aligned} -1 &= -x^2 - 4x \\ -1 + \underline{(-4)} &= -(x^2 + 4x + \underline{4}) \\ -5 &= -(x+2)^2 \\ y &= -(x+2)^2 + 5 \end{aligned} \quad \begin{aligned} -5 &= -(x+2)^2 \\ \sqrt{5} &= \sqrt{(x+2)^2} \\ \pm\sqrt{5} &= x+2 \\ -2 \pm \sqrt{5} & \end{aligned}$$



→ Vertex
Standard form: $y = -(x+2)^2 + 5$

Zeros: $-2 \pm \sqrt{5}$

Vertex: $(-2, 5)$ Axis of symmetry: $x = -2$

$y\text{-int } (0, 1)$



Min or Max (circle one) and value: 5

3. Find all real zeros of the polynomial function $f(x) = \underline{2x^4 - 2x^3 - 60x^2}$ algebraically. Specify the multiplicity of each zero. (4 pts)

$$0 = x^4 - x^3 - 30x^2$$

$$x^2(x^2 - x - 30)$$

$$x^2(x+5)(x-6)$$

0 DR -5 6

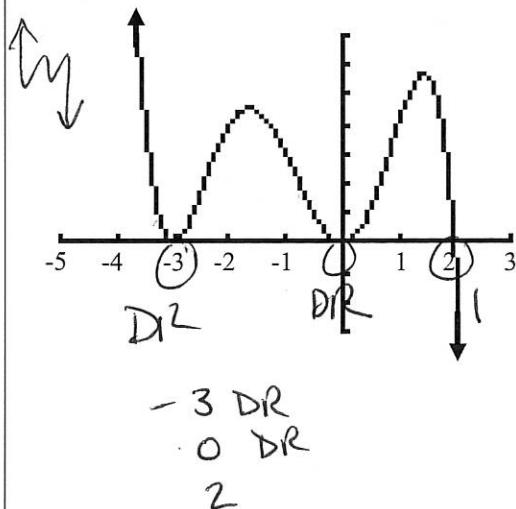
ZEROS: $\{0 \text{ DR}, -5, 6\}$

$\begin{matrix} & 1 \\ \text{mult } 2 & \end{matrix}$ $\begin{matrix} & 1 \\ \text{mult } 1 & \end{matrix}$

4. Find a polynomial function of degree 3 that has zeros -1 and 2 . (4 pts)

$$\begin{aligned} & \underbrace{(x+1)}_2 \underbrace{(x-2)}_1 \\ & (x-2)(x^2 + 2x + 1) \\ & \cancel{x^3 + 2x^2 + 2x^3} \\ & \cancel{\cancel{x^3}} \cancel{\cancel{x^2}} \cancel{\cancel{x}} \\ & \begin{array}{c|cc|c} x & x^3 & 2x^2 & x \\ \hline -2 & -2x^2 & -4x & -2 \end{array} \\ & f(x) = x^3 - 3x^2 + 4x \\ & x^3 - 3x^2 + 4x = f(x) \end{aligned}$$

5. Construct a polynomial function that could have the graph shown below. Make sure to expand all factors and simplify the polynomial. (6 pts)



*negative.
Factored Polynomial $-(x^2(x+3)^2(x-2)) = f(x)$

Simplified Polynomial $f(x) = -x^5 - 4x^4 + 3x^3 + 18x^2$

→ HINT: use LCT to check your answer!

$(x+3)^2 = x^2 + 6x + 9$

~~00~~ $\begin{array}{c|cc|c} x^2 & 6x & 9 \\ \hline x & x^3 & 6x^2 & 9x \\ \hline -2 & -2x^2 & -12x & -18 \end{array}$

$x^2(x^3 + 4x^2 - 3x - 18)$

$f(x) = x^5 + 4x^4 - 3x^3 - 18x^2$

$-x^5 - 4x^4 + 3x^3 + 18x^2$

8-11: Sketch the graph of the polynomial function

$$f(x) = \frac{1}{4}(x^4 - 2x^3 - 4x^2 + 8x). \quad (10 \text{ pts total})$$

$a > 0$
even

6. State the tail behavior using the leading coefficient test.

(2pt)

rise left
rise right

7. Find the zeros *algebraically*.

$$\underline{x^4 - 2x^3 - 4x^2 + 8x}$$

$$x^3(x-2) - 4x(x-2)$$

$$(x^3 - 4x)(x-2) \quad \{0, -2, 2\} \text{ DR}$$

$$x(x^2 - 4)(x-2)$$

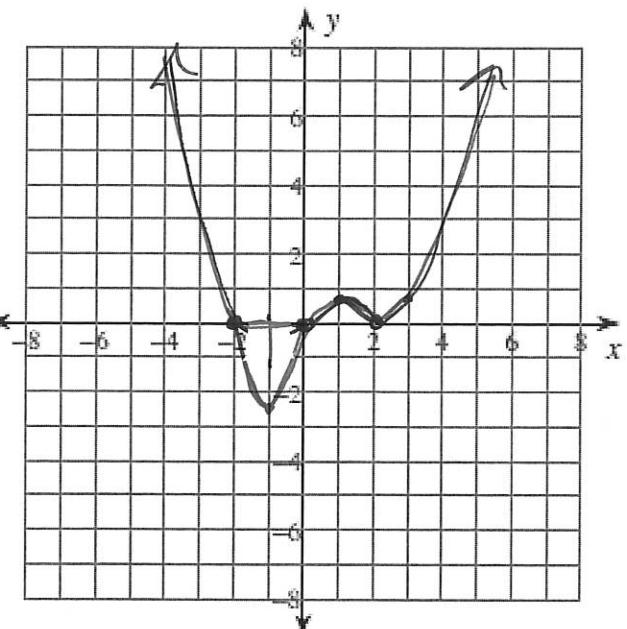
$$x(x+2)(x-2)(x-2)$$

8. Specify additional points to plot.

x	y
1	$\frac{1}{4}(1-2-4+8) = .75$
-1	$\frac{1}{4}(1+2+4-8) = \frac{1}{4}(-1) = -2.25$
3	3.75

9. Plot all from above points and sketch a

smooooooth curve through them with the proper behavior at the zeros.



BONUS: (2 points for correct answer)

Write the standard vertex form equation of the parabola that has zeros $(-5, 0)$ and $(1, 0)$ and maximum value $y = 18$.

$$(x+5)(x-1)$$

$$x^2 - x + 5x - 5$$

$$x^2 + 4x - 5 = 0$$

~~$$(x^2 + 4x + 4) = 5 + 4$$~~

$$(x+2)^2 = 9 \quad \text{NOPE!}$$

$$(x+2)-9=0$$

~~max should be 18 so mult by -2~~

$$-2((x+2)-9=0)$$

$$-2(x+2)^2 + 18$$

