Honors Pre-Calculus **Final Exam Review #4: 3.4-3.5** **Mr. Chancellor**

Name: SOLUTIONS Date: ____________

Problems 1-6: Solve each exponential or logarithmic equation algebraically (i.e. without a calculator). Write your answer in exact form, then compute it as a decimal rounded to .xxx.

1. $7^{5x} = 317$

$\log_7(7^{5x}) = \log_7 317$

$5x = \dfrac{\log 317}{\log 7}$ $\therefore \boxed{x = \dfrac{1}{5}\dfrac{\log 317}{\log 7}}$

$x \approx 0.592$

2. $-2e^{2x} + 60 = 12$

$-2e^{2x} = -48$

$e^{2x} = 24$

$2x = \ln 24$

$\boxed{x = \frac{1}{2}\ln 24 \quad x \approx 1.589}$

3. $\ln\sqrt{x-2} = 3$

$e \qquad e$

$\sqrt{x-2} = e^3$

$x - 2 = e^6$

$\boxed{x = 2 + e^6} \approx 405.429$

4. $6 - 3\ln x = 12$

$-3\ln x = 6$

$\ln x = -2$

$e \qquad e$

$x = e^{-2} = \dfrac{1}{e^2} \approx 0.135$

5. $\log x + \log(x-7) = -\log 12$

$\log x + \log(x-7) + \log 12 = 0$

$\log[x(x-7)(12)] = 0$

$10 \qquad 10$

$12x(x-7) = 1$

$12x^2 - 84x - 1 = 0$

$x = \dfrac{84 \pm \sqrt{84^2 - 4(12)(-1)}}{24} = \dfrac{84 \pm 8\sqrt{111}}{24}$

$\uparrow$

$\dfrac{21 \pm 2\sqrt{111}}{6}$

$\dfrac{21 + 2\sqrt{111}}{6}$ $\boxed{\approx 7.012}$

6. $\ln x + \ln(x+5) = \ln 6$

$\ln[x(x+5)] = \ln 6$

$x^2 + 5x = 6$

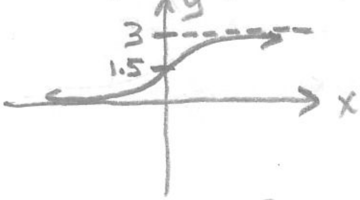
$x^2 + 5x - 6 = 0$

$(x+6)(x-1) = 0$

$x = \cancel{-6}, 1$ $\quad x = 1$

7. On your calculator, graph $f(x) = \dfrac{3}{1+e^{-5x}}$.

What are the horizontal asymptotes and what is the y-intercept? Make a sketch of the graph.



H.A.: $y = 3,\ y = 0$

$f(0) = \dfrac{3}{2}$

8. Solve $\dfrac{12}{1+3e^{-2x}} = 10$

$12 = 10(1 + 3e^{-2x})$

$\dfrac{6}{5} = 1 + 3e^{-2x}$

$\dfrac{1}{5} = 3e^{-2x}$

$\dfrac{1}{15} = e^{-2x}$

$\to 15 = e^{2x}$

$\ln 15 = 2x$

$\boxed{x = \frac{1}{2}\ln 15}$

$\approx \boxed{1.354}$

Problems 9-10: The number of hits a new web site received follows the model $H(t) = 4080e^{0.2988t}$, where H is the number of hits per month and t is the number of months since it started.

9. Find the number of hits it received in its third month.

$H(3) = 4080e^{0.2988(3)}$

≈ 9999.1

$\approx \boxed{10000 \text{ hits}}$

10. Find the number of months it will take to receive 5,000,000 hits.

$5000000 = 4080e^{0.2988t}$

$1225.490 = e^{0.2988t}$

$t = \dfrac{1}{0.2988} \ln(1225.490)$

$\boxed{t \approx 23.8 \text{ months}}$ (about 2 years)

Problems 11-12: The population of Flagstaff was 26,117 in 1970 and 52,894 in 2000. Suppose the population growth during that period was exponential according to the model $P(t) = a \cdot e^{kt}$.

11. Find a and k and write down the final population model. Round any decimals to .xxxx.

$P(t) = ae^{kt}$ Let $t=0$ be 1970

$\boxed{a = 26117}$

$52894 = 26117e^{30k}$

$2.025 = e^{30k}$

$\therefore k = \dfrac{1}{30} \ln 2.025$

$\boxed{k \approx 0.0235}$

12. In what year did the population exceed 45,000?

$45000 = 26117e^{0.0235t}$

$1.723 = e^{0.0235t}$

$t = \dfrac{1}{0.0235} \ln 1.723$

$t \approx 23.15$ years (past 1970)

$\therefore \boxed{1993}$

13. The number of households in the U.S. that own widgets showed logistic growth from 1980 to 1999 according to $H(t) = \dfrac{91.86}{1 + 22.96e^{-0.4t}}$, where H is the number of households (in millions) and t is the number of years since 1980. In what year were there 86 million households that owned a widget?

$86 = \dfrac{91.86}{1+22.96e^{-0.4t}}$

$86(1+22.96e^{-0.4t}) = 91.86$

$1.068 = 1 + 22.96e^{-0.4t}$

$0.068 = 22.96e^{-0.4t}$

$0.00297 = e^{-0.4t}$

$t = -\dfrac{1}{0.4} \ln 0.00297$

$t \approx 14.5$ years past 1980

$\boxed{1994}$