

Name: \_\_\_\_\_

**Lessons A.2.1 & A.2.2:** Simplify each expression involving exponents. Your answer should have only POSITIVE exponents.

<p>1. <math>\frac{2x^4 y^{-4} z^{-3}}{3x^2 y^{-3} z^4}</math></p> <p><math>\frac{2x^4 y^3}{3x^2 y^4 z^4 z^3} = \frac{2x^2}{3yz^7}</math></p>	<p>2. <math>\frac{3x^2 y^2}{2x^{-1} \cdot 4yx^2}</math></p> <p><math>\frac{3x^2 y^2}{2 \cdot 4 x^2 y} = \frac{3xy}{8}</math></p>	<p>3. <math>\frac{2m^{-4}}{(2m^{-4})^3}</math></p> <p><math>\frac{2m^{-4}}{8m^{-12}} = \frac{m^8}{4}</math></p>
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<p>4. <math>\frac{(2pm^{-1}q^0)^{-4} \cdot 2m^{-1}p^3}{2pq^2}</math></p> <p><math>\frac{2^{-4} p^{-4} m^4 q^0 \cdot 2m^{-1} p^3}{2pq^2} = \frac{m^3}{16p^2 q^2}</math></p>	<p>5. <math>\frac{(2hj^2k^{-2} \cdot h^4 j^{-1} k^4)^0}{2h^{-3} j^{-4} k^{-2}}</math></p> <p><math>\frac{1}{2} = \frac{h^3 j^4 k^2}{2}</math> (cancels to 1)</p>	<p>6. <math>x^{m-n}(x^{n+m} - x^{n-m} + x^n)</math></p> <p><math>x^{m-n+n+m} = x^{2m}</math>  <math>-x^{m-n+n-m} = -x^0 = -1</math>  <math>x^{m-n+n} = x^m</math></p> <p><math>x^{2m} + x^m - 1</math></p>
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<p>7. <math>\left(\frac{x^{-2}y^{-1}}{x^{-1}y^{-2}}\right)^{-3}</math></p> <p><math>\frac{x^6 y^3}{x^3 y^6} = \frac{x^3}{y^3}</math></p>	<p>8. <math>\left(\frac{a^0}{b}\right)^{-2} \left(\frac{a}{b^{-2}}\right)^{-2}</math></p> <p><math>\frac{1}{b^{-2}} \cdot \frac{a^{-2}}{b^4} = \frac{b^2}{a^2 b^4}</math></p> <p><math>= \frac{1}{a^2 b^2}</math></p>	
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**Lessons A.2.4 & A.2.5:** Simplify each expression involving radicals. Make sure to apply the absolute value restriction as necessary. If an expression does not represent a real number, write "not real".

<p>9. <math>(-\sqrt{11})^2</math></p> <p><math>(-\sqrt{11})(-\sqrt{11}) = 11</math></p>	<p>10. <math>\sqrt[4]{y^4}</math> even even odd</p> <p><math> y </math></p>	<p>11. <math>(\sqrt[4]{ y })^4</math></p> <p><math> y </math> (already positive)</p>	<p>12. <math>\sqrt[3]{x^3}</math></p> <p><math>x</math></p>	<p>13. <math>\sqrt[4]{-144}</math></p> <p>even roots can't have negative radicand (not real)</p>	<p>14. <math>-\sqrt[4]{144}</math></p> <p><math>-(\sqrt{12}) = -2\sqrt{3}</math></p>
<p>15. <math>-\sqrt{0.04}</math></p> <p><math>-\sqrt{\frac{4}{100}} = -\frac{2}{10} = -0.2</math></p>	<p>16. <math>\sqrt[3]{-5^3}</math></p> <p><math>-5</math></p>	<p>17. <math>\sqrt{-5^2}</math></p> <p>OH NO!</p> <p>not real</p>	<p>18. <math>\sqrt{(-x)^2}</math></p> <p><math>= \sqrt{x^2} =  x </math></p>	<p>19. <math>\sqrt[3]{x^6}</math></p> <p><math>(x^6)^{1/3} = x^2</math></p>	<p>20. <math>\sqrt[6]{x^6}</math> even even odd</p> <p><math> x </math></p>

21. $\sqrt[3]{135}$ $\begin{array}{c} 5 \quad 27 \\ \downarrow \quad \downarrow \\ 3^3 \end{array}$ $\sqrt[3]{27 \cdot 5}$ $\boxed{3\sqrt[3]{5}}$	22. $\sqrt[3]{20} \cdot \sqrt[3]{14}$ $\sqrt[3]{\cancel{2} \cdot \cancel{2} \cdot 5 \cdot \cancel{2} \cdot 7}$ $2^3 \sqrt[3]{5 \cdot 7} = \boxed{2 \cdot \sqrt[3]{35}}$	23. $\frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3}$ $\boxed{2\sqrt{3}}$
24. $\sqrt[3]{-16a^3b^8}$ $\sqrt[3]{(-8)(2)a^3b^6b^2}$ $\boxed{-2ab^2 \cdot \sqrt[3]{2b^2}}$	25. $\sqrt[4]{128x^7y^7}$ $\begin{array}{c} 2 \quad 2 \quad 2 \quad 2 \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ 2^4 \end{array}$ $\sqrt[4]{2^4 \cdot 2^3 \cdot x^4 \cdot x^3 \cdot y^4 \cdot y^3}$ $\boxed{2xy \cdot \sqrt[4]{8x^3y^3}}$	
26. $\sqrt[3]{\frac{2}{9}}$ $\frac{\sqrt[3]{2}}{\sqrt[3]{9}}$ $\frac{\sqrt[3]{2}}{\sqrt[3]{3^2} \cdot \sqrt[3]{3}}$ $\boxed{\frac{\sqrt[3]{6}}{3}}$	27. $\frac{\sqrt[4]{5}}{4\sqrt[4]{27}}$ $\frac{\sqrt[4]{5}}{4\sqrt[4]{3^3} \cdot \sqrt[4]{3}}$ $\frac{\sqrt[4]{15}}{4(3)} = \boxed{\frac{\sqrt[4]{15}}{12}}$	28. $\sqrt[4]{\frac{2y^6}{3x^3}}$ $\frac{\sqrt[4]{2y^6} \cdot \sqrt[4]{3^3x}}{\sqrt[4]{3x^3} \cdot \sqrt[4]{3^3x}}$ $\frac{\sqrt[4]{54xy^6}}{3x} = \boxed{\frac{y \sqrt[4]{54xy^2}}{3x}}$

**ANSWERS**

1. $\frac{2x^2}{3yz^7}$	2. $\frac{3xy}{8}$	3. $\frac{m^8}{4}$	4. $\frac{m^3}{16p^2q^2}$	5. $\frac{h^3j^4k^2}{2}$	6. $x^{2m} + x^m - 1$
7. $\frac{x^3}{y^3}$	8. $\frac{1}{a^2b^2}$				
9. 11	10. $ y $				
11. $y$	12. $x$	13. not real	14. $-2\sqrt{3}$	15. $-0.2$	16. $-5$
17. not real	18. $ x $	19. $x^2$	20. $ x $		
21. $3 \cdot \sqrt[3]{5}$	22. $2 \cdot \sqrt[3]{35}$	23. $2\sqrt{3}$	24. $-2ab^2 \cdot \sqrt[3]{2b^2}$	25. $2xy \cdot \sqrt[4]{8x^3y^3}$	
26. $\frac{\sqrt[3]{6}}{3}$	27. $\frac{\sqrt[4]{15}}{12}$	28. $\frac{y \cdot \sqrt[4]{54xy^2}}{3x}$			



Name: Key**Lessons A.2.6 & A.2.7:** Simplify each expression involving radicals. Make sure to use the absolute value restriction when appropriate.

$$1. \sqrt{18} + \sqrt{24} - \sqrt{54}$$

$$\sqrt{9 \cdot 2} + \sqrt{4 \cdot 6} - \sqrt{9 \cdot 6}$$

$$3\sqrt{2} + 2\sqrt{6} - 3\sqrt{6}$$

$$\boxed{3\sqrt{2} - \sqrt{6}}$$

$$2. \sqrt[4]{96} + \sqrt[4]{486} - \sqrt[4]{32}$$

$$6 \sqrt[4]{16} + 8 \sqrt[4]{81} - 2 \sqrt[4]{16}$$

$$2\sqrt[4]{6} + 3\sqrt[4]{6} - 2\sqrt[4]{2}$$

$$\boxed{5\sqrt[4]{6} - 2\sqrt[4]{2}}$$

$$3. \sqrt{7\sqrt{2}} + \sqrt{2\sqrt{7}}$$

$$\frac{\sqrt{14} \cdot 7}{2 \cdot 7} + \frac{\sqrt{14} \cdot 2}{7 \cdot 2}$$

$$\frac{7\sqrt{14} + 2\sqrt{14}}{14} = \boxed{\frac{9\sqrt{14}}{14}}$$

$$4. \sqrt[4]{96} - \sqrt[4]{\frac{2}{27}}$$

$$\sqrt[4]{16 \cdot 6} - \frac{\sqrt[4]{3}}{\sqrt[4]{3^3}} = \frac{\sqrt[4]{6}}{3}$$

$$2\sqrt[4]{6} - \frac{\sqrt[4]{6}}{3}$$

$$\frac{6\sqrt[4]{6}}{3} - \frac{\sqrt[4]{6}}{3} = \boxed{\frac{5\sqrt[4]{6}}{3}}$$

$$5. \sqrt{\frac{3}{8}} \left( \sqrt{\frac{3}{4}} + \frac{2}{\sqrt{3}} \right)$$

$$\frac{\sqrt{9}}{\sqrt{32}} + \frac{2\sqrt{3}}{\sqrt{24}} = \frac{3\sqrt{2}}{4\sqrt{2}\sqrt{2}} + \frac{2\sqrt{3}\sqrt{6}}{2\sqrt{6}\sqrt{6}}$$

$$\frac{3\sqrt{2}}{8} + \frac{2\sqrt{18}}{12} = \frac{9\sqrt{2}}{24} + \frac{4\sqrt{18}}{24}$$

$$\frac{9\sqrt{2} + 4 \cdot 3\sqrt{2}}{24} = \frac{9\sqrt{2} + 12\sqrt{2}}{24} = \frac{21\sqrt{2}}{24} = \boxed{\frac{7\sqrt{2}}{8}}$$

$$6. \sqrt{2a^2b^4} + \sqrt{8a^2b^4}$$

$$\frac{1}{2}ab^2\sqrt{2} + \frac{2}{2}ab^2\sqrt{2}$$

$$\boxed{3/2ab^2\sqrt{2}}$$

$$7. \sqrt{6x} + \sqrt{\frac{2x\sqrt{3}}{3\sqrt{3}}} \sqrt{\frac{3x}{2}\sqrt{2}}$$

$$\sqrt{6x} + \frac{\sqrt{6x}}{3} + \frac{\sqrt{6x}}{2}$$

$$\frac{6\sqrt{6x}}{6} + \frac{2\sqrt{6x}}{6} + \frac{3\sqrt{6x}}{6}$$

$$\boxed{\frac{5\sqrt{6x}}{6}}$$

$$8. \sqrt{10y}(\sqrt{2y^5} - \sqrt{5y})$$

$$\sqrt{20y^6} - \sqrt{50y^2}$$

$$4.5 \sqrt{20y^6} - 25.2 \sqrt{50y^2}$$

$$\boxed{2y^3\sqrt{5} - 5y\sqrt{2}}$$

$$9. (\sqrt{5} + 2)^2(\sqrt{5} + 2)$$

$$\sqrt{25} + 2\sqrt{5} + 2\sqrt{5} + 4$$

$$5 + 4\sqrt{5} + 4$$

$$\boxed{4\sqrt{5} + 9}$$

$$10. (3\sqrt{11} - \sqrt{10})^2(3\sqrt{11} - \sqrt{10})$$

$$9 \cdot 11 - 3\sqrt{110} - 3\sqrt{110} + 10$$

$$99 - 6\sqrt{110} + 10$$

$$\boxed{109 - 6\sqrt{110}}$$

$$11. \frac{1}{6 + \sqrt{3}} \frac{6 - \sqrt{3}}{6 - \sqrt{3}}$$

$$\frac{6 - \sqrt{3}}{36 - 3} = \boxed{\frac{6 - \sqrt{3}}{33}}$$

$$12. \frac{\sqrt{6}}{\sqrt{2} + \sqrt{3}} \frac{\sqrt{2} - \sqrt{3}}{\sqrt{2} - \sqrt{3}}$$

$$\frac{\sqrt{12} - \sqrt{18}}{2 - 3} = \frac{2\sqrt{3} - 3\sqrt{2}}{-1}$$

$$= \boxed{3\sqrt{2} - 2\sqrt{3}}$$

$$13. \frac{2\sqrt{7} - \sqrt{3}}{\sqrt{7} + \sqrt{3}} \frac{\sqrt{7} - \sqrt{3}}{\sqrt{7} - \sqrt{3}}$$

$$\frac{2 \cdot 7 - 2\sqrt{21} - \sqrt{21} + 3}{7 - 3}$$

$$\boxed{\frac{17 - 3\sqrt{21}}{4}}$$

$$14. \text{Reciprocal of } \frac{\sqrt{5} + 1}{2}$$

$$\frac{2}{\sqrt{5} + 1} \frac{\sqrt{5} - 1}{\sqrt{5} - 1} = \frac{2\sqrt{5} - 2}{5 - 1}$$

$$\frac{2\sqrt{5} - 2}{4} = \boxed{\frac{\sqrt{5} - 1}{2}}$$

Lessons A.2.8 & A.2.9: Simplify each expression. If not possible, state why.

15. $36^{\frac{3}{2}}$ $(\sqrt{36})^3$ $6^3 = 216$	16. $-16^{\frac{3}{4}}$ $-(\sqrt[4]{16})^3$ $-2^3 = -8$	17. $(-32)^{\frac{3}{5}}$ $(-2^5)^{\frac{3}{5}}$ $-2^{-3} = -\frac{1}{2^3} = -\frac{1}{8}$	18. $(-16)^{\frac{5}{4}}$ ← not real <del><math>(-2^4)^{\frac{5}{4}}</math></del> <del><math>-2^5 = -32</math></del>
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Convert into exponential form having only positive exponents.

19. $\sqrt[3]{a^{-2}b}$ $a^{-2/3}b^{1/3}$ <del><math>\frac{b^{1/3}}{a^{2/3}}</math></del> $\frac{b^{1/3}}{a^{2/3}}$	20. $\sqrt[3]{8b^6c^{-4}}$ $(8b^6c^{-4})^{1/3}$ $8^{1/3}b^{6/3}c^{-4/3}$ $\frac{2b^2}{c^{4/3}}$	21. $\frac{1}{\sqrt[6]{x^4y^{-12}}}$ $\frac{1}{(x^4y^{-12})^{1/6}}$ $\frac{1}{x^{4/6}y^{-2}} = \frac{y^2}{x^{2/3}}$
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22. $\frac{\sqrt[4]{x} \cdot \sqrt[6]{x} \div \sqrt[3]{x}}{x^{1/3}}$ $\frac{x^{1/4} \cdot x^{1/6}}{x^{1/3}} = \frac{x^{3/12} \cdot x^{2/12}}{x^{4/12}} = \frac{x^{5/12}}{x^{4/12}} = x^{1/12}$ $x^{1/12}$	23. $\left(\left(b^{1/2}\right)^{-2/3}\right)^{3/4}$ $\frac{1}{2} \cdot \frac{-2}{3} \cdot \frac{3}{4} = -\frac{1}{4}$ $b^{-1/4} = \frac{1}{b^{1/4}}$
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Convert into simplified radical form.

24. $\sqrt[3]{8} \cdot \sqrt[6]{8}$ $(2^3)^{1/3} (2^3)^{1/6}$ $2^{3/3} 2^{3/6} = 2^{1+1/2} = 2^{3/2} = 2\sqrt{2}$ $2^{4/2} = 2^2 = 4$	25. $\frac{\sqrt[5]{27^3}}{\sqrt[5]{9^2}}$ $\frac{(27)^{3/5} = (3^3)^{3/5}}{(9)^{2/5} = (3^2)^{2/5}}$ $\frac{3^{9/5}}{3^{4/5}} = \frac{3^9}{3^4} = \frac{3^5}{1} = 3^1 = 3$	26. $\sqrt[6]{8^3} \cdot \sqrt[4]{4^2}$ $(8)^{3/6} \cdot (4)^{2/4}$ $(2^3)^{1/2} \cdot (2^2)^{1/2}$ $2^{3/2} \cdot 2^{2/2} = 2^{5/2} = 2^{2+1/2} = 2^2 \cdot 2^{1/2} = 4\sqrt{2}$
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ANSWERS

1. $3\sqrt{2} - \sqrt{6}$	2. $5\sqrt[4]{6} - 2\sqrt[4]{2}$	3. $\frac{9\sqrt{14}}{14}$	4. $\frac{5\sqrt[4]{6}}{3}$	5. $\frac{7\sqrt{2}}{8}$
6. $3 a b^2\sqrt{2}$	7. $\frac{5\sqrt{6x}}{6}$	8. $2y^3\sqrt{5} - 5y\sqrt{2}$	9. $9 + 4\sqrt{5}$	
10. $109 - 6\sqrt{110}$	11. $\frac{6 - \sqrt{3}}{33}$	12. $3\sqrt{2} - 2\sqrt{3}$	13. $\frac{17 - 3\sqrt{21}}{4}$	14. $\frac{\sqrt{5} - 1}{2}$
15. 216	16. -8	17. $-\frac{1}{8}$	18. not real (even root of neg. num.)	19. $\frac{b^{1/3}}{a^{2/3}}$
20. $\frac{2b^2}{c^{4/3}}$	21. $\frac{y^2}{x^{2/3}}$	22. $x^{1/12}$	23. $\frac{1}{b^{1/4}}$	
24. 4	25. 3	26. $4\sqrt{2}$		