Honors Pre-Calculus **Final Exam Review #1: 2.1-2.4** **Mr. Chancellor**

Name: SOLUTIONS Date: _____

1. Convert the quadratic function $f(x) = 3x^2 - 12x - 8$ to standard form.

$3(x^2 - 4x + 4) - 8 - 12$

$\left(\frac{4}{2}\right)^2$

$\boxed{3(x-2)^2 - 20}$

2. Write the standard form equation of the parabola that has zeros $(-3, 0)$ and $(3, 0)$ and maximum value $y = 18$.

Axis of symmetry @ midpoint of zeros. ∴ vertex @ $(0, 18)$

$f(x) = ax^2 + 18$

$0 = 9a + 18 \qquad a = -2$

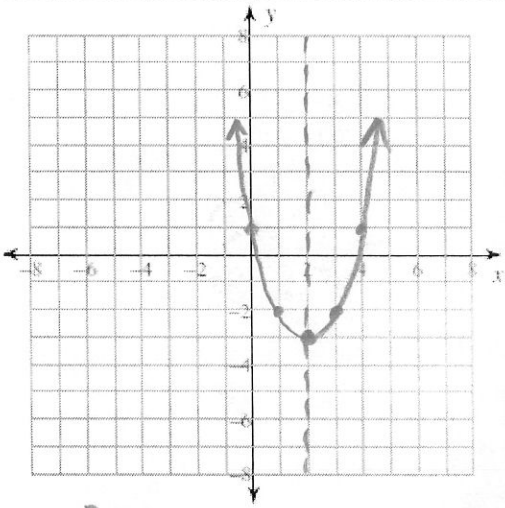
$\boxed{f(x) = -2x^2 + 18}$

3. Convert the quadratic function $g(x) = x^2 - 4x + 1$ to standard form and specify the **vertex**, **axis of symmetry**, **value of the maximum or minimum**, and find its **zeros**. algebraically. Graph the function and identify all of the above on the graph.

$g(x) = (x^2 - 4x + 4) + 1 - 4$

$\left(\frac{4}{2}\right)^2$

$\boxed{g(x) = (x-2)^2 - 3}$



Standard form: _____

Vertex: $(2, -3)$ Axis of symmetry: $x = 2$

Min or Max (circle one) and value: $y = -3$

OPENS UP

$(x-2)^2 - 3 = 0$

$(x-2)^2 = 3$

$x - 2 = \pm\sqrt{3}$

Zeros: $2 \pm \sqrt{3} \approx 3.732, 0.268$

4. A company that makes widgets developed a relationship between profit (P) and dollars spent on advertising (x) as $P = 800 + 10x - 0.25x^2$. ***Algebraically*** find the number of advertising dollars that maximizes profit and specify the maximum profit.

Maximum profit occurs @ the vertex : $\left(-\frac{b}{2a}, P\left(-\frac{b}{2a}\right)\right)$

$x_v = \frac{-10}{2(-0.25)} = 20$

$y_v = 800 + 10(20) \quad 0.25(20)^2 = 900$

$\boxed{\text{MAX. PROFIT} = \$900 \text{ AT } x = \$20}$

5. Find all real zeros of the polynomial function $f(x) = 2x^4 - 2x^3 - 40x^2$ ***algebraically***. Specify the multiplicity of each zero.

$2x^2(x^2 - x - 20) = 0$

$2x^2(x-5)(x+4) = 0$

$\boxed{x = 0\ (DZ),\ 5,\ -4}$

6. Find a polynomial function of degree 3 that has zeros -2 and 1.

$\hookrightarrow \therefore$ factors $(x-(-2))$ & $(x-1)$

$(x+2)^2(x-1)$ OR $(x+2)(x-1)^2$

$(x^2+4x+4)(x-1)$

$x^3 - x^2 + 4x^2 - 4x + 4x - 4$

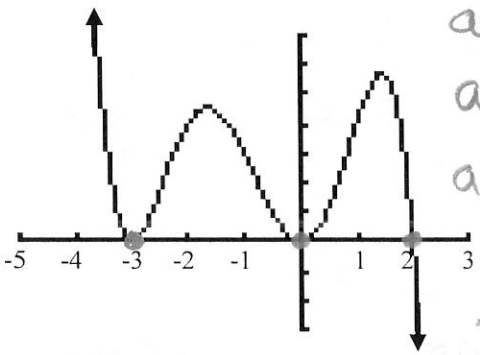
$x^3 + 3x^2 - 4$

$(x+2)(x^2-2x+1)$

$x^3 + 2x^2 - 2x^2 - 4x + x + 2$

$x^3 - 4x + x + 2$

7. Construct a polynomial function of the form $a_n x^n + a_{n-1}x^{n-1} + \cdots a_1 x + a_0$ that could have the graph shown below. Make sure to expand all factors and simplify the polynomial.



$a(x+3)^2 \cdot x^2 \cdot (x-2)$ with $\uparrow$ $\downarrow$ tails

$a(x^2+6x+9)(x^3-2x^2)$

$a(x^5 - 2x^4 + 6x^4 - 12x^3 + 9x^3 - 18x^2)$

$a(x^5 + 4x^4 - 3x^3 - 18x^2)$ $\quad a < 0$

$\boxed{-x^5 - 4x^4 + 3x^3 + 18x^2}$

8-11: Sketch the graph of the polynomial function $f(x) = \frac{1}{4}\left(x^4 - 2x^3 - 4x^2 + 8x\right)$.

8. State the tail behavior using the ***leading coefficient test***.

n is even & $a_n > 0$ $\therefore$ $\uparrow$ $\uparrow$

9. Find the zeros ***algebraically***.

$x^4 - 2x^3 - 4x^2 + 8x = 0$

$x(x^3 - 2x^2 - 4x + 8) = 0$

$x(x^2-4)(x-2) = 0$

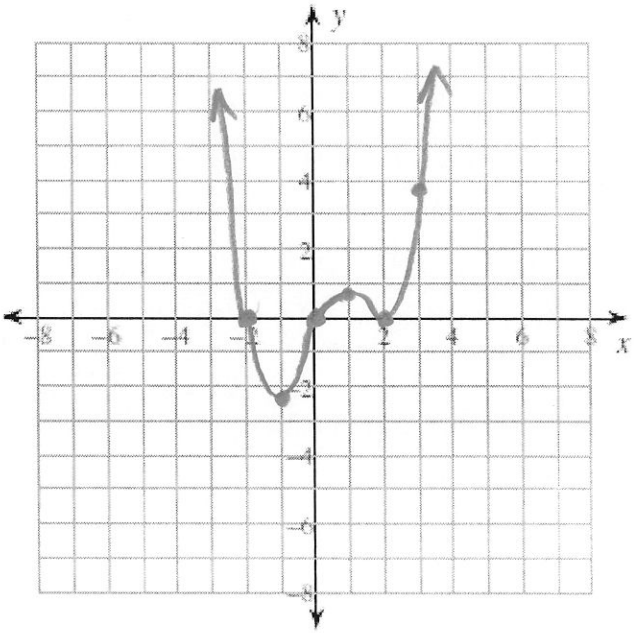
$x(x+2)(x-2)^2 = 0$

$\boxed{x = 0,\ -2,\ 2\ (D.Z.)}$

10. Specify additional points to plot.

x	y
-3	18.75
-1	-2.25
1	0.75
3	3.75

11. Plot all from above points and sketch a smooth curve through them with the proper behavior at the zeros.



12. Divide using ***synthetic division***.

$$\frac{5x^3+6x-8}{x+2}$$

$$\begin{array}{r|rrrr} -2 & 5 & 0 & 6 & -8 \\ & & -10 & 20 & -52 \\ \hline & 5 & -10 & 26 & -60 \end{array}$$

$$\boxed{5x^2-10x+26-\frac{60}{x+2}}$$

13. Find $f(-6)$ for $f(x)=10x^3-22x^2-3x+4$ using ***synthetic substitution***.

$$\begin{array}{r|rrrr} -6 & 10 & -22 & -3 & 4 \\ & & -60 & 492 & -2934 \\ \hline & 10 & -82 & 489 & \boxed{-2930} \end{array}$$

14. Determine if $x=-4$ is a zero of the function $f(x)=x^3-28x-48$. If so, ***factor*** the polynomial function completely.

$$\begin{array}{r|rrr|r} -4 & 1 & 0 & -28 & -48 \\ & & -4 & 16 & 48 \\ \hline & 1 & -4 & -12 & 0 \end{array}$$

$\leftarrow$ is a zero

$x^2-4x-12$

$(x-6)(x+2)$

$$\boxed{(x+4)(x-6)(x+2)=f(x)}$$

15. Find all zeros of $f(x)=x^4-4x^3-15x^2+58x-40$ given that $(x-5)$ and $(x+4)$ are factors.

$$\begin{array}{r|rrrrr} 5 & 1 & -4 & -15 & 58 & -40 \\ & & 5 & 5 & -50 & 40 \\ \hline -4 & 1 & 1 & -10 & 8 & 0 \checkmark \\ & & -4 & 12 & -8 & \\ \hline & 1 & -3 & 2 & 0 \checkmark & \end{array}$$

x^2-3x+2 $\quad (x-2)(x-1)=0$

$x=2,1$

$\therefore$ zeros: $\boxed{5,-4,2,1}$

16. Multiply $\left(\sqrt{3}+i\sqrt{15}\right)\left(\sqrt{3}-i\sqrt{15}\right)$

$3-i^2\cdot 15=3+15$

$=18$

17. Divide $\dfrac{6-7i}{1-2i}\cdot\dfrac{1+2i}{1+2i}$ (hint: conjugate)

$\dfrac{6+12i-7i-14i^2}{1+5}$

$\boxed{\dfrac{20+5i}{6}}$

18. Simplify i^{8269}

$4\overline{)8269}=2067\text{ R }1$

$\therefore \boxed{i^{8269}=i}$

19. Solve $2x^2+6x+10=0$

$x^2+3x+5=0$

$x=\dfrac{-3\pm\sqrt{9-4(5)}}{2}$

$=\boxed{\dfrac{-3\pm i\sqrt{11}}{2}}$