

Name: SOLUTIONS

Date: _____

1. List all possible rational zeros of
- $f(x)$
- .

$$f(x) = 2x^5 + 8x^4 - 15x^3 + 22x^2 + 5x - 8$$

$$\begin{array}{r} \pm 1, \pm 8, \pm 4, \pm 2 \\ \hline \pm 1, \pm 2 \end{array}$$

$$\boxed{\pm 1, \pm \frac{1}{2}, \pm 8, \pm 4, \pm 2}$$

2. List all possible numbers of positive and negative zeros using Descartes' Rule of Signs for
- $g(x) = 3x^3 + 2x^2 + x + 3$
- .

$$g(x): + + +$$

$$g(-x): - + -$$

0 + real zeros

3 or 1 - real zeros

3. Find all zeros of
- $f(x) = 2x^4 - 9x^3 + 8x^2 - x - 60$
- , given that
- $x = 1 - 2i$
- is a zero. (Hint: what other zero can you know immediately?)

recall: $x^2 + bx + c = x^2 - (\text{sum of zeros})x + \text{product of zeros}$

If $1 - 2i$ is a zero, $1 + 2i$ is a zero: sum = 2 product = 5

$\therefore x^2 - 2x + 5$ is a factor

$$\begin{array}{r} 2x^2 - 5x - 12 \xrightarrow{\quad} 2x^2 - 8x + 3x - 12 = 0 \\ x^2 - 2x + 5 \overline{)2x^4 - 9x^3 + 8x^2 - x - 60} \\ - (2x^4 - 4x^3 + 10x^2) \\ \hline - 5x^3 - 2x^2 - x \\ - (-5x^3 + 10x^2 - 25x) \\ \hline - 12x^2 + 24x - 60 \\ - (-12x^2 + 24x - 60) \\ \hline 0 \end{array}$$

$$(2x+3)(x-4) = 0$$

$$x = -\frac{3}{2}, 4$$

$$\therefore \boxed{\text{zeros: } -\frac{3}{2}, 4, 1 \pm 2i}$$

4. Given
- $f(x) = 2x^3 + 9x^2 + 14x + 5$
- , a) List all possible rational zeros, b) search for any rational zeros, and c) solve the depressed equation.

$$(a) \frac{\pm 1, \pm 5}{\pm 1, \pm 2} \Rightarrow \pm 1, \pm 5, \pm \frac{1}{2}, \pm \frac{5}{2}$$

$$(b) \cancel{\boxed{2}} \quad \begin{array}{r} 2 \ 9 \ 14 \ 5 \\ \cancel{2} \ \cancel{11} \ \cancel{25} \\ \hline 2 \ 11 \ 25 \end{array}$$

$$\cancel{\boxed{2}} \quad \begin{array}{r} 2 \ 9 \ 14 \ 5 \\ -2 \ -7 \ -7 \\ \hline 2 \ 7 \ 7 \end{array}$$

$$\cancel{\boxed{2}} \quad \begin{array}{r} 2 \ 9 \ 14 \ 5 \\ 10 \\ \hline 2 \ 19 \end{array}$$

$$\cancel{\boxed{2}} \quad \begin{array}{r} 2 \ 9 \ 14 \ 5 \\ -10 \ 5 \\ \hline 2 \ -1 \ 19 \end{array}$$

$$\cancel{\boxed{2}} \quad \begin{array}{r} 2 \ 9 \ 14 \ 5 \\ 1 \ 5 \\ \hline 2 \ 10 \ 19 \end{array}$$

$$\cancel{\boxed{-\frac{1}{2}}} \quad \begin{array}{r} 2 \ 9 \ 14 \ 5 \\ -1 \ -4 \ -5 \\ \hline 2 \ 8 \ 10 \ 0 \end{array}$$

$$2x^2 + 8x + 10 = 0$$

$$x^2 + 4x + 5 = 0$$

$$x = -4 \pm \sqrt{16 - 4(5)} \quad \boxed{x = -4 \pm \sqrt{16 - 20}}$$

$$\begin{aligned} x &= -\frac{4 \pm 2i}{2} \\ &= -2 \pm i \end{aligned}$$

$$\boxed{\text{Zeros: } -\frac{1}{2}, -2 \pm i}$$

Problems 5-6: Graph each rational function. Provide all of the information requested.

5. $f(x) = \frac{x^2 - 5x + 4}{x^2 - 4} = \frac{(x-4)(x-1)}{(x+2)(x-2)}$

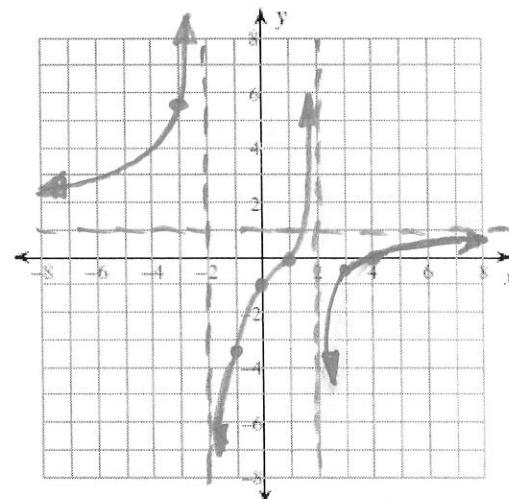
x	y
-3	28/5
-1	10/-3
3	-2/5
5	4/21

Domain: $(-\infty, \infty)$ except ± 2

y-intercept: $(0, -1)$

Zeros: $4, 1$

Vertical asymptotes: $x = \pm 2$



Horizontal or Slant asymptote and its equation:

$$y = \frac{1}{1} = 1$$

6. $f(x) = \frac{x^2 - x + 1}{x - 1}$

x	y
2	3

Domain: $(-\infty, \infty)$ except $x = 1$

y-intercept: $(0, -1)$

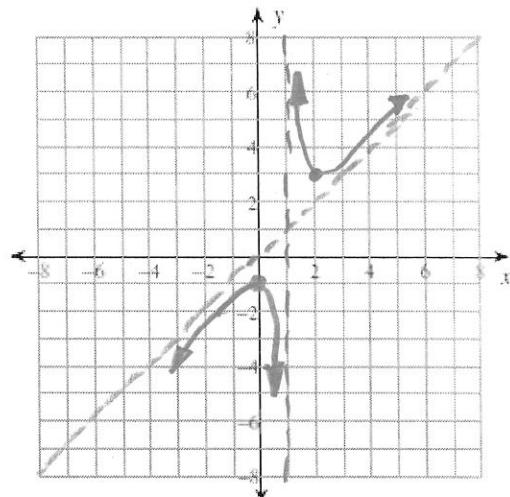
Zeros: NO REAL ZEROS $x = \frac{1 \pm \sqrt{1-4(1)}}{2}$

Equation of vertical asymptotes: $x = 1$

Horizontal or Slant asymptote and its equation:

$$\begin{array}{r} 1 \ 1 \ -1 \ 1 \\ \underline{-} 1 \ 0 \ 1 \\ 1 \ 0 \ 1 \end{array}$$

$$x + \frac{1}{x-1} = f(x) \therefore \text{Slant asymptote } y = x$$



Problems 7-8: Solve each inequality. Write the solution in the proper interval format and graph the solution on a number line using proper symbols.

7. $x^2 - 2x + 9 > 17$

$$x^2 - 2x - 8 > 0$$

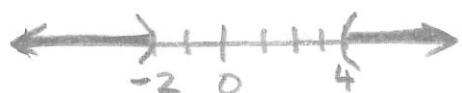
$$(x-4)(x+2) > 0$$

$$(-\infty, -2) \quad (-2, 4) \quad (4, \infty)$$

TEST: -3 0 5

POLY: + - +
SIGN

SOLUTION:
$$(-\infty, -2) \cup (4, \infty)$$



8. $\frac{2}{x+5} < \frac{1}{x-3}$

$$\frac{2}{x+5} - \frac{1}{x-3} < 0$$

$$\frac{2(x-3) - (x+5)}{(x+5)(x-3)} < 0 \Rightarrow \frac{x-11}{(x+5)(x-3)} < 0$$

KEY POINTS: $x = 11, -5, -3$

$$(-\infty, -5) \quad (-5, -3) \quad (-3, 11) \quad (11, \infty)$$

TEST -6 -4 0 12

NUM. - - - +

DEN. + - + +

FRAC. - + - +

$$(-\infty, -5) \cup (-3, 11)$$



9. Given the graph of $f(x)$, solve the inequality $f(x) \geq 0$. Write your solution in interval notation and graph it on a number line with the proper symbols.

$$(-\infty, -3] \cup [-1, 3] \cup [5, 7]$$

