

Information About the Final Exam

- The final exam is cumulative, covering **Appendix A (A.1-A.5)** and **Chapter 1**.
- All problems will be multiple choice with one correct answer out of 4.
- You may use your calculator for the whole final exam. It will be reset before the test!
- The study guide below has all of the topics we have covered and you will be working selected review problems. The problems may not cover every concept, so a good strategy is to study those topics you do not immediately know.
- Each day we will have a warm-up review and a set of problems to work. You will turn in a packet with all warm-ups and problems on the day of the final exam.
- The final exam accounts for 20% of your total semester average. While your grade is not just a numeric computation, your average does play a huge part in determining your grade.
- You **MUST** be present on the specified day and time. There will **NOT** be any make-ups!
- **BRING YOUR GRAPHING CALCULATOR!!!**

APPENDIX A.1: Properties of Real Numbers

- **Hierarchy of real numbers** and the subsets for integers, whole numbers, and natural numbers.
- Recognize **rational and irrational numbers** when represented as decimals.
- Express **intervals** on the number line graphically using interval notation with [] instead of • and () instead of ○ .
- Write intervals using [] and () and inequality notation and how to convert words into intervals.
- Determine the **distance** between two points using absolute value.
- Definition of **expression** and what it means to **simplify an expression**.
- Use the order of operations to simplify an expression
- State the "rules of algebra" given "For all real numbers a, b, c, \dots " and one of the properties. For example, "State the transitive property for all real numbers a, b, c ."
- Identify the properties that were used to simplify an algebraic statement.
- Add, subtract, and multiply signed numbers
- Divide integers and fractions using the definition of division: multiplication by the reciprocal

APPENDIX A.2: Exponents and Radicals

- Know the exponent rules for any real numbers a and b and **integers** m and n :

$a^m \cdot a^n = a^{m+n}$	$(a \cdot b)^m = a^m \cdot b^m$	$(a^m)^n = a^{m \cdot n}$	$\frac{a^m}{a^n} = a^{m-n}$	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$a^0 = 1$
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- Simplify expressions containing exponents according to a variety of instructions. For example, know how to express an answer with only positive exponents or without any fractions.
- Definition of **scientific notation**: $m \times 10^n$, where m is in the interval $[1, 10)$ and n is an integer.
- Convert from decimal to scientific notation and vice versa. Know how to move the decimal point and to set the exponent properly depending on the size of the number.
- **Significant digits**: All non-zero numbers are significant. Zeros are significant only when they do something other than place the decimal. For example, 5.0 has 2 SD whereas 50 has 1 SD; 4008 has 4 SD whereas 4000 has 1 SD.
- Know the definition of the **n^{th} root of a number**: the solution to the equation $x^n = b$.
- Understand the structure of a radical: $\sqrt[\text{index}]{\text{radicand}}$
- Simplify radicals by looking for factors of the radicand that are perfect n^{th} powers.
- Apply the absolute value restriction to a simplified radical: when the index is even and the resulting root has a variable with an odd exponent. **NEVER** restrict an odd-index radical (e.g. cube root).

- Rationalize the denominator of a radical by finding a multiplier that "completes the perfect n^{th} power".
- Add radicals by finding like radicals (same index and radicand).
- Multiply binomials containing radicals using the distributive property (sometimes called FOIL).
- Rationalize a binomial denominator by multiplying by the conjugate.
- Understand the relationship between rational exponents and radicals: $b^{\frac{m}{n}} = (\sqrt[n]{b})^m = \sqrt[n]{b^m}$.
- Convert a radical into exponential form and vice versa.
- Simplify expressions with rational exponents using the relationship above.
- Simplify expressions containing radicals by first converting into exponential form, using exponent rules to simplify the new expression, then converting back to radical form at the end.
- Simplify expressions containing real number exponents using the standard rules of exponents.

APPENDIX A.3: Polynomials and Factoring

- Simplify a polynomial and write in standard form (decreasing degree of lowest alphabet variable)
- State the degree of a monomial and the degree of a polynomial
- Multiply polynomials using the distributive property
- Recognize special products to make the multiplication process simpler
- Factor a **difference of squares**: $a^2 - b^2 = (a + b)(a - b)$
- Factor a **sum or difference of cubes**: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ and $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
- Factor a **perfect square trinomial**: $a^2 \pm 2ab + b^2 = (a \pm b)^2$
- Factor a **trinomial**: $ax^2 + bx + c$ using the $a \cdot c$ method
- Factor a **trinomial in quadratic form**: $a(*)^2 + b(*) + c$ using the $a \cdot c$ method
- Factor a **polynomial by grouping** terms to form common factors so that $(\) + (\) + \dots + (\)$ becomes $(\) \cdot (\) \cdot \dots \cdot (\)$. Look for difference of squares, perfect square trinomials, and other groupings that form common factors.

APPENDIX A.4: Rational Expressions

- Definitions: **domain** of an expression; equivalent expressions
- State the domain of a rational expression or an expression involving roots.
- Simplify a rational expression by prime factoring the numerator and denominator, dividing common factors, and stating restrictions.
- Understand the difference between the domain of a rational expression and restrictions stated on an equivalent expression.
- "Simplify then multiply" rational expressions and state any restrictions on the simplified form
- Turn a division problem into multiplication by the reciprocal
- Find the least common denominator (LCD) of two or more rational expressions and convert all fractions to have the LCD.
- Simplify a **complex fraction** using "method A" (simplify num & den, then divide by multiplying by the reciprocal or "method B" (multiply by LCD/LCD, where the LCD is for **all** denominators in the complex fraction).
- Simplify a complex fraction involving rational exponents by factoring out the **smallest** exponent and simplifying the resulting expression.
- Simplify a complex fraction involving rational exponents by multiplying by a factor that makes all exponents whole numbers.

APPENDIX A.5: Solving Equations

- Know the meaning of **equation** and **solve an equation**.
- Solve linear equations using the properties of equality to create an equivalent equation of the form $x = c$.
- Be able to recognize when there is no solution and when the solution is all the reals (from an identity).
- Know how to eliminate all fractions by multiplying the entire equation by the LCD.
- **CAUTION:** Multiplying by an LCD with variables is not guaranteed to create an equivalent equation, so you **MUST** check all potential solutions in the **ORIGINAL** equation.
- Know the definition of the **roots of an equation**.
- Solve polynomial equations by converting them to standard form (expression = 0), factoring the expression, setting each factor equal to zero, and solving for each root.
- Know how to derive the **quadratic formula** by completing the square on $ax^2 + bx + c = 0$.
- Solve equations of the form $ax^2 + bx + c = 0$ by completing the square or the quadratic formula. **NOTE:** You must be able to demonstrate how to solve by completing the square.
- Be able to solve equations that are in quadratic form: $a(*)^2 + b(*) + c = 0$, where * represents an expression.
- Solve radical equations by isolating the radical, squaring both sides, and isolating the variable.
- **CAUTION:** Squaring both sides of a radical equation is not guaranteed to make an equivalent equation. You must check all potential solutions in the **ORIGINAL** equation. Make sure to state if a potential solution is really an extraneous root.

CHAPTER 1: Functions and Their Graphs

Section 1.1: The coordinate plane

- Distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Midpoint formula: $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$
- Use the distance formula to find the length of a segment or to verify that three points form a right triangle.
- Use the midpoint formula to find the midpoint of a line segment or to perform another task that requires the midpoint.
- Know the coordinate form of **transformations** and how to transform a point in the coordinate plane.
 - Translation: $(x, y) \rightarrow (x + a, y + b)$
 - Reflection in x -axis: $(x, y) \rightarrow (x, -y)$
 - Reflection in y -axis: $(x, y) \rightarrow (-x, y)$
 - Reflection in origin: $(x, y) \rightarrow (-x, -y)$

Section 1.2: Graphs of equations

- Know the definition of an **equation in two variables** and the **solution** to an equation in two variables.
- Sketch the graph of an equation in two variables using the point plotting method.
- Understand the meaning of **x -intercept** and **y -intercept** and be able to find them algebraically for an equation in two variables.
- Know the meaning of **symmetry** and be able to determine from a graph or algebraically if an equation has x -axis symmetry, y -axis symmetry, origin symmetry, or no symmetry.

Section 1.3: Graphs of lines and linear equations

- Know the definition of the **slope** of a line and how to find the slope through two points. Pay particular attention to the special cases of a horizontal line (slope = 0) and a vertical line (slope undefined).
- Know **slope-intercept**: $y = mx + b$.
- Be able to graph the equation of a line in slope-intercept form or a vertical line in " x -form".
- Be able to write the equation of a line in slope-intercept form when given two points on the line.

- Determine if two lines are **parallel** or **perpendicular** and know how to create the equation of a line parallel or perpendicular to a given line passing through a given point.
- Solve word problems related to linear equations and interpret the results in the context of the problem.

Section 1.5: Analyzing functions by their graphs

- In general, know **how** to answer the "7 questions", though I do not expect you to recite the actual questions.
- Be able to identify the **domain** and **range** of a relationship (function or not) directly from its graph.
- Determine if a graph represents a function using the **vertical line test**.
- Find the **zeros** of a function **algebraically** and **graphically** using the calculator (**CALC, 2:zero**).
- Estimate the intervals where a function is **increasing**, **decreasing**, or **constant graphically**.
- Determine all **maxima** and **minima** of a function graphically using the calculator (**CALC, 3:minimum** or **CALC, 4:maximum**)
- Be able to create a function from a word problem and find its maximum or minimum using the calculator.

Section 1.6: Graphs of parent functions

- Know how to graph all of the parent functions using the **3 point method**. Recognize that the squaring, cubic, square root, and absolute value functions all have (0, 0) and (1, 1) in common. The third point depends on the function itself.
- Be able to graph the greatest integer function with correct use of • and ◦ on the endpoints of the steps.

Section 1.7: Transformations of parent functions

- Know how to convert a given function into standard graphing form: $g(x) = a f [b(x-c)] + d$ where b is factored out.
- Know the effects of a , b , c , and d .
 - a causes a **vertical stretch** if $|a| > 1$, a **vertical shrink** if $0 < |a| < 1$, and a **reflection in the x-axis** if $a < 0$.
 - b causes a **horizontal shrink** if $|b| > 1$, a **horizontal stretch** if $0 < |b| < 1$, and a **reflection in the y-axis** if $b < 0$.
 - c causes a **horizontal shift** to the right if $c > 0$ and to the left if $c < 0$. Remember, $x + c = x - (-c)$.
 - d causes a **vertical shift** up if $d > 0$ and down if $d < 0$.
- Be able to graph a transformed function using the three step procedure:
 1. Write the function in standard graphing form.
 2. Locate the "new origin" by applying any **translations** to (0, 0).
 3. Plot additional points **from the new origin** using the basic relationship (x, y) on $f(x)$ becomes $\left(\frac{1}{b}x, ay\right)$ on $g(x)$.
- Know how to convert a word description of transformations into function notation.
- Be able to describe the transformations of a parent function $f(x)$ when given a function $g(x)$.

Practice Problems

You DO NOT need to turn in this packet when you turn in your work

A.1 and A.2

P. A24-A26 33, 35, 37, 41, 43, 45, 47, 53, 55, 63, 67, 69, 77, 83, 85, 89, 97,
105, 107, 109, 117

A.3 and A.4

P. A36 – A37 75, 77, 91, 103, 113, 139, 155, 161, 171, 201, 207

P. A46 – A47 35, 37, 53, 55, 59, 65, 73, 79

A.5 and A.6

P. A60 – A61 21, 25, 27, 39, 41, 53, 65, 73, 85, 95, 115, 119, 123, 129, 131,
139, 149, 153

P. A69 – A70 49, 59 – 65 Odd

1.1 – 1.7

P. 116 – 119 5, 25, 49, 59, 61, 63, 65, 71, 73, 87, 89, 97, 99, 101-107 odd,
115-119 odd, 123, 127