Name: _____

1. How many odd numbers less than 10,000 can you make using only the digits 0, 3, 4, and 7?	**2.** How many license plates having 3 symbols (letters and digits) can you make that have at least one digit?
3. Evaluate $\frac{1000!}{995!}$	**4.** Evaluate ${}_{100}P_2$.
5. In how many ways can you arrange the letters in the word UNCOPYRIGHTABLE?	**6.** How many seven letter words can you make from the letters in AMBIDEXTROUSLY?
7. In how many ways can you arrange the letters in SESQUIPEDALIOPHOBIA?	**8.** A box contains 8 ***identical*** red pens, 6 ***identical*** blue pens, and 10 other ***different*** pens. How many different ways are there of giving one pen to each of 24 students?

9. 24 points lie randomly on the circumference of a circle. How many inscribed pentagons can you make having these points as vertices?

10. Suppose there are 54 sophomores and 34 juniors at SPA and that the prom committee will have 10 members, split equally among these two classes. How many different committees can you make?

11. How many 5-card poker hands can you deal having exactly three of one type of card (e.g. three aces)?

12. In the card game "Bridge", each person is dealt 13 cards. How many different hands can be dealt having exactly 5 diamonds?

13. Expand $(x - y)^7$ using the binomial theorem.

14. Expand $(2x^3 - 3xy^2)^5$ using the binomial theorem.

15. Find the 3rd term of the expansion $(4a^6 + 6b^3)^7$.

Name: Solutions

1. How many odd numbers less than 10,000 can you make using only the digits 0, 3, 4, and 7?

1D: [2] = 2 (odd)

2D: can't be 0 → [3]·[2] = 6

3D: [3]·[4]·[2] = 24

4D: [3]·[4]·[4]·[2] = 96

128

OR [4]·[4]·[4]·[2] =

2. How many license plates having 3 symbols (letters and digits) can you make that have at least one digit?

3D: DDD

2D: LDD, DLD, DDL

1D: DLL, LDL, LLD

$(10\cdot10\cdot10) + 3(10\cdot10\cdot26) + 3(10\cdot26\cdot26)$

$1000 + 7800 + 20280$

$= 29{,}080$

3. Evaluate $\frac{1000!}{995!}$ $1000\cdot999\cdot998\cdot997\cdot996\cdot995!$ / $995!$

9.9003×10^{14}

4. Evaluate ${}_{100}P_2$.

$\frac{100!}{98!} = 100\cdot99 = 9900$

5. In how many ways can you arrange the letters in the word UNCOPYRIGHTABLE? $n=15$

(no repeats)

$15! = 1.308\times10^{12}$

(OR ${}_{15}P_{15}$)

6. How many seven letter words can you make from the letters in AMBIDEXTROUSLY? $n=14$

${}_{14}P_7$ $\frac{14!}{7!} = 17{,}297{,}280$

7. In how many ways can you arrange the letters in SESQUIPEDALIOPHOBIA?

$\frac{19!}{2!\,2!\,3!\,2!\,2!\,2!}$

S E I P A O

$= 6.336\times10^{14}$

8. A box contains 8 ***identical*** red pens, 6 ***identical*** blue pens, and 10 other ***different*** pens. How many different ways are there of giving one pen to each of 24 students?

$\frac{24!}{8!\,6!} = 2.137\times10^{16}$

ways of arranging the reds & blues that you can't count more than once because they are the same

9. 24 points lie randomly on the circumference of a circle. How many inscribed pentagons can you make having these points as vertices?

pentagon → 5 points

$_{24}C_5 = 42{,}504$

10. Suppose there are 54 sophomores and 34 juniors at SPA and that the prom committee will have 10 members, split equally among these two classes. How many different committees can you make?

Soph Jun

$_{54}C_5 \cdot {}_{34}C_5$

$= 8.8 \times 10^{11}$

11. How many 5-card poker hands can you deal having exactly three of one type of card (e.g. three aces)?

$13 \cdot {}_4C_3 \cdot {}_{48}C_2 = 58{,}656$

13 types · $_4C_3$ ways to choose 3 of a kind · $_{48}C_2$ ways to choose 2 other cards

12. In the card game "Bridge", each person is dealt 13 cards. How many different hands can be dealt having exactly 5 diamonds?

5 ♦ 8 non ♦

$_{13}C_5 \cdot {}_{39}C_8 = 7.918 \times 10^{10}$

13. Expand $(x-y)^7$ using the binomial theorem.

from Δ → 1 7 21 35 35 21 7 1

$1x^7y^0 \quad 7x^6y^1 \quad 21x^5y^2 \quad 35x^4y^3 \quad 35x^3y^4 \quad 21x^2y^5 \quad 7xy^6 \quad 1xy^7$

$x^7 - 7x^6y + 21x^5y^2 - 35x^4y^3 + 35x^3y^4 - 21x^2y^5 + 7xy^6 - y^7$

14. Expand $(2x^3 - 3xy^2)^5$ using the binomial theorem.

Δ → 1 5 10 10 5 1

$1(2x^3)^5(\quad)^0 - \frac{5(2x^3)^4(3xy^2)^1}{5\cdot 16\cdot 3} + \frac{10(2x^3)^3(3xy^2)^2}{10\cdot 8\cdot 9} - \frac{10(2x^3)^2(3xy^2)^3}{10\cdot 4\cdot 27} + \frac{5(2x^3)^1(3xy^2)^4}{5\cdot 2\cdot 81} - \frac{1(\quad)(3xy^2)^5}{243}$

$32x^{15} - 240x^{12}xy^2 + 720x^9x^2y^4 - 1080x^6x^3y^6 + 810x^3x^4y^8 - 243x^5y^{10}$

$32x^{15} - 240x^{13}y^2 + 720x^{11}y^4 - 1080x^9y^6 + 810x^7y^8 - 243x^5y^{10}$

15. Find the 3rd term of the expansion $(\overset{x}{4a^6} + \overset{y}{6b^3})^7$.

Δ → 1 7 21 35 35 21 7 1

r=0 r=1 ★ r=2

3rd term → r=2

$_nC_r\, x^{n-r} y^r \qquad n=7 \quad r=2$

$_7C_2 \cdot x^{7-2}y^2$

$21 \cdot (4a^6)^5 (6b^3)^2$

$21 \cdot 1024a^{30} \cdot 36b^6$

$774144a^{30}b^6$