Honors Pre-calculus **TEST REVIEW 9.6-9.7** **Mr. Chancellor**

Name: SOLUTIONS **Section:** ________

SECTION 9.6: Fundamental counting principles, permutations, and combinations. Make sure to show proper "P" and "C" setup. You must show a complete setup to receive credit.

1. How many odd numbers less than 10,000 can you make using only the digits 0, 3, 4, and 7?

1D CAN'T BE 0 $\boxed{2} = 2$

2D $\boxed{3}\boxed{2} = 6$

3D $\boxed{3}\boxed{4}\boxed{2} = 24$

4D $\boxed{3}\boxed{4}\boxed{4}\boxed{2} = 96$

$\boxed{128}$

2. How many license plates having 3 symbols (letters and digits) can you make that have at least one digit?

PLACES FOR DIGIT

1D $\boxed{10}\boxed{26}\boxed{26} \times 3 = 20280$

Places for letter

2D $\boxed{10}\boxed{10}\boxed{26} \times 3 = 7800$

3D $\boxed{10}\boxed{10}\boxed{10} = 1000$

$\boxed{29{,}080}$

3. Evaluate $\dfrac{1000!}{995!} = 1000 \cdot 999 \cdot 998 \cdot 997 \cdot 996$

9.9003×10^{14}

4. Evaluate ${}_{100}P_2$. $= \dfrac{100!}{98!} = 100 \cdot 99$

$= 9900$

5. In how many ways can you arrange the letters in the word UNCOPYRIGHTABLE? $n=15$

(No repeats)

$\boxed{15!} \approx \boxed{1.308 \times 10^{12}}$

6. How many seven letter words can you make from the letters in AMBIDEXTROUSLY? $n=14$

${}_{14}P_7 = \dfrac{14!}{7!} = 17{,}297{,}280$

7. In how many ways can you arrange the letters in SESQUIPEDALIOPHOBIA?

$$\frac{19!}{2!\,2!\,3!\,2!\,2!\,2!}$$

S E I P A O

$= 6.336 \times 10^{14}$

8. A box contains 8 ***identical*** red pens, 6 ***identical*** blue pens, and 10 other ***different*** pens. How many different ways are there of giving one pen to each of 24 students?

$$\frac{24!}{\underbrace{8!\,6!}}$$

arrangements

$= 2.137 \times 10^{16}$

ways of arranging the reds & blues that you can't count more than once b/c they are same.

9. 24 points lie randomly on the circumference of a circle. How many inscribed pentagons can you make having these points as vertices?

${}_{24}C_5$

$= 42{,}504$

10. Suppose there are 54 sophomores and 34 juniors at SPA and that the prom committee will have 10 members, split equally among these two classes. How many different committees can you make?

$\boxed{{}_{54}C_5} \cdot \boxed{{}_{34}C_5}$

SOPH. JUN.

$= 8.8 \times 10^{11}$

11. How many 5-card poker hands can you deal having exactly three of one type of card (e.g. three aces)?

$\boxed{13} \cdot \boxed{{}_4C_3} \cdot \boxed{{}_{48}C_2}$ ← ways to choose any two other types

$= \boxed{58656}$

↑ TYPES; ↑ WAYS TO CHOOSE 3 FROM EACH GROUP OF 4

12. In the card game "Bridge", each person is dealt 13 cards. How many different hands can be dealt having exactly 5 diamonds?

$\boxed{{}_{13}C_5} \cdot \boxed{{}_{39}C_8} = \boxed{7.918 \times 10^{10}}$

↑ WAYS TO PICK 5 ♦; ↑ WAYS TO PICK 8 NON-♦

SECTION 9.7: Probability. Express all probabilities as **simplified fractions**.

13. An experiment consists of drawing two cards simultaneously from a standard 52-card deck.

How many elements are in the sample space?

${}_{52}C_2 = 1326$

Find the probability of each event:

both are hearts

$\frac{{}_{13}C_2}{1326} = \frac{1}{17}$

both are jacks

$\frac{{}_4C_2}{1326} = \frac{1}{221}$

neither is a spade (39 "non spades")

$\frac{{}_{39}C_2}{1326} = \frac{19}{34}$

14. A bag has 4 blue rocks, 5 red rocks, and 6 green rocks. An experiment consists of drawing two rocks simultaneously and noting their colors.

How many elements are in the sample space?

${}_{15}C_2 = 105$

Find the probability of each event:

both are red

$\frac{{}_5C_2}{105} = \frac{10}{105} = \frac{2}{21}$

one red, one blue

$\frac{\boxed{5} \cdot \boxed{4}}{105} = \frac{4}{21}$

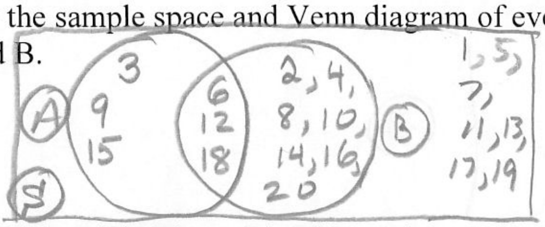
at least one green

$P(1G) + P(2G)$

$\frac{6 \cdot 9 + {}_6C_2}{105} = \frac{69}{105} = \frac{23}{35}$

15. An experiment consists of rolling a 20-sided die and the number (n) recorded. Define two events, **A: *n* is a multiple of 3** and **B: *n* is even**.

Draw the sample space and Venn diagram of events A and B.



Find P(A), P(B), P(A ∩ B), P(A ∪ B)

$P(A) = \frac{6}{20} = \frac{3}{10}$ $\quad P(B) = \frac{10}{20} = \frac{1}{2}$

$P(A \cap B) = \frac{3}{20}$ $\quad P(A \cup B) = \frac{6}{20} + \frac{10}{20} - \frac{3}{20} = \frac{13}{20}$

Are events A & B *mutually exclusive*? Explain.

No, because there is a non-empty intersection

Are events A & B *independent*? Explain.

$P(A) \cdot P(B) = \frac{3}{20} = P(A \cap B)$

∴ INDEPENDENT

16. A bag contains 3 pennies and 5 dimes. An experiment consists of selecting two coins, ***one at a time without replacement***.

Find the number of items in the sample space.

$\boxed{8} \cdot \boxed{7} = 56$

Find the probability of each event:

Both coins are dimes

$\frac{\boxed{5} \cdot \boxed{4}}{56} = \frac{20}{56} = \frac{5}{14}$

The first is a penny

$\frac{3}{8}$ (DON'T CARE ABOUT 2ND PICK)

The second is a penny

$\underbrace{PP \text{ or } DP}_{\text{mutually excl.}}$ $\quad \frac{3 \cdot 2}{56} + \frac{5 \cdot 3}{56} = \frac{21}{56} = \frac{3}{8}$

At least one is a dime

$= 1 - P(PP)$

$= 1 - \frac{3 \cdot 2}{56} = \frac{50}{56} = \frac{25}{28}$

17. In baseball, the probability that Ted will get a hit is 0.406; the probability that Lou will get a hit is 0.340; and the probability Bob will get a hit is 0.200. Find the probability of each event after one at-bat each.

All three get a hit

$(0.406)(0.340)(0.200) = 0.028$

Ted gets a hit, but Lou and Bob do not

$(0.406)(1-0.340)(1-0.200)$

$= 0.214$

None of them gets a hit

$(1-0.406)(1-0.340)(1-0.200)$

$= 0.314$

At least one batter gets a hit.

$= 1 - P(\text{none})$

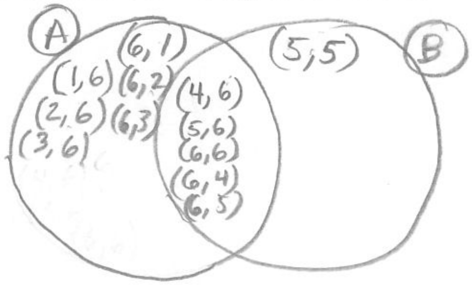
$= 1 - 0.314$

$= 0.686$

18. An experiment consists of rolling a blue die and a white die. The outcome is an ordered pair (b, w). Define two events, **A: at least one die is a 6** and **B: their sum is greater than 9**.

10, 11, 12

Draw the Venn diagram of events A and B.



Write the sets $A \cap B$ and $A \cup B$

$A \cap B = \{(4,6), (5,6), (6,6), (6,4), (6,5)\}$

$A \cup B = \{\text{all of them in } A, B, A \cap B\}$

Find $P(A)$, $P(B)$, $P(A \cap B)$, $P(A \cup B)$

$P(A) = \frac{11}{36}$ $\quad P(B) = \frac{6}{36} = \frac{1}{6}$

$P(A \cap B) = \frac{5}{36}$ $\quad P(A \cup B) = \frac{11}{36} + \frac{6}{36} - \frac{5}{36}$

$= \frac{12}{36} = \frac{1}{3}$

Are events A & B ***mutually exclusive***? Explain.

No – intersection exists.

Are events A & B ***independent***? Explain.

$P(A) \cdot P(B) = \frac{11}{216} \neq \frac{5}{36}$

$\therefore$ DEPENDENT