SOLUTIONS

Honors Pre-calculus **Test Review 9.1-9.3** **Mr. Chancellor**

Work each of the problems on your own paper. This will be DUE the next class period.

1. Find a formula for the nth term of the sequence $17, 7, -3, \ldots$, then find a_{1000}.

Arithmetic $d = -10$

$a_n = 17 - 10(n-1) = \boxed{27 - 10n}$

$a_{1000} = 17 - 10(999)$

$= -9973$

2. Find a_{365} of the arithmetic sequence if $a_{10} = 41$ and $a_{15} = 61$.

$61 = 41 + d(15-10) \therefore d = 4$

$a_{365} = 61 + 4(365-15)$

$= 1461$

3. Insert four arithmetic means between 11 and 35.

$11, \_, \_, \_, \_, 35$

$35 = 11 + d(6-1) \therefore d = \frac{24}{5}$

$\boxed{15.8,\ 20.6,\ 25.4,\ 30.2}$

4. How many terms are in the sequence $44, 36, 28, \ldots, -380$?

$-380 = 44 - 8(n-1)$

$-424 = -8(n-1)$

$53 = n-1$

$n = 54$

5. How many $\boxed{\text{multiples of 3}}$ are there between 100 and 100,000? ↳ divisible by 3

$102, 105, 108, \ldots, 99{,}999$

$99999 = 102 + 3(n-1)$

$33299 = n-1$

$\boxed{n = 33{,}300}$

6. Find a_{10} for the sequence $-5, 4, \cancel{-\frac{4}{5}, \frac{-16}{25}}, \ldots$

Geometric : $r = -\frac{4}{5}$

$\boxed{-5, 4, -\frac{16}{5}, \ldots}$

$a_{10} = -5\left(-\frac{4}{5}\right)^9$

$= -5\left(\frac{-262144}{1953125}\right)$

$= +\frac{262144}{390625} \approx 0.671$

7. Find a_7 of the geometric sequence if $a_2 = -\frac{2}{3}$ and $a_5 = \frac{16}{81}$.

$\frac{16}{81} = -\frac{2}{3} r^3$

$r^3 = \frac{\cancel{16}^{\,8}}{\cancel{81}_{\,27}} \cdot \left(\frac{-\cancel{3}}{\cancel{2}}\right) = \frac{-8}{27}$

$\therefore r = -\frac{2}{3}$

$a_7 = \left(\frac{16}{81}\right)\left(-\frac{2}{3}\right)^2$

$= \frac{64}{729}$

8. Insert four geometric means between 1 and 2.

$1, \_, \_, \_, \_, 2$

$2 = 1 \cdot r^5$

$r = \sqrt[5]{2}$

$\boxed{\sqrt[5]{2}, \sqrt[5]{4}, \sqrt[5]{8}, \sqrt[5]{16}}$

$\boxed{2^{1/5}, 2^{2/5}, 2^{3/5}, 2^{4/5}}$

9. A pile of bricks has 85 bricks in the bottom row, 79 in the second row, 73 in the third row, etc. until there is only 1 brick in the last row. How many bricks are in the 12th row and how many total rows are there?

85, 79, 73, ... Arithmetic, $d = -6$

$a_{12} = 85 - 6(11) = 19$

$1 = 85 - 6(n-1)$

$-84 = -6(n-1)$

$14 = n - 1$ $\therefore$ $\boxed{15 \text{ rows}}$

10. A new car worth \$32,500 new decreases in value by 25% per year. How much is it worth after 7 years?

$32500,\ 32500(0.75),\ 32500(0.75)^2, \ldots$

$a_8 = 32500(0.75)^7$ ← number of depreciations that have occurred after 7 years

$\approx \boxed{\$4338.23}$

11. As all SPA music students well know, there are 12 steps in the chromatic scale from the A below middle C to the next higher A. The frequency of the A below middle C is 220 Hz (1 Hz = 1 cycle/sec). The frequency of the next higher A is, therefore, 440 Hz. Given that the notes in between these two As form a geometric sequence, find the common ratio.

$440 = 220 \cdot r^{12}$ (12 steps → 13 terms)

$2 = r^{12}$

$\therefore r = \sqrt[12]{2} = \boxed{2^{1/12} \approx 1.0595}$

12. Express the finite series $2+4+6+\cdots+1000$ in sigma notation. Do not evaluate the series.

Arithmetic, $d = 2$; $a_n = 2 + 2(n-1)$

$a_n = 2n$

$1000 = 2 + 2(n-1)$ $\therefore n = 500$

$$\boxed{\sum_{n=1}^{500} 2n}$$

13. Express the infinite series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$ in sigma notation. Do not evaluate the series.

$$\boxed{\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1}}$$

14. Find the 100th partial sum of the series $100 + 98 + 96 + \cdots$

Arithmetic, $d = -2$

$a_{100} = 100 - 2(99) = -98$

$S_{100} = \frac{100(100 + (-98))}{2}$

$= \boxed{100}$

15-16: Find S_{10} if the series $24 + 12 + \cdots$ is:

15. Arithmetic : $d = -12$

$a_{10} = 24 - 12(9) = -84$

$S_{10} = \frac{10(24 + (-84))}{2} = -300$

16. Geometric : $r = \frac{1}{2}$

$S_{10} = \frac{24\left(1 - \left(\frac{1}{2}\right)^{10}\right)}{1 - \frac{1}{2}} = 48\left(\frac{1023}{1024}\right)$

$= \boxed{\frac{3069}{64}}$

17. Evaluate $\sum_{k=0}^{12}\left(-\frac{1}{2}\right)^k$

13 terms

Geometric

$r=\frac{1}{2}$

$a_1=1$

$$S_{13}=\frac{1\left(1-\left(-\frac{1}{2}\right)^{13}\right)}{1-\left(-\frac{1}{2}\right)}=\frac{2}{3}\left(1+\frac{1}{8192}\right)$$

$$=\frac{2}{3}\,\frac{8193}{8192}$$

$$=\boxed{\frac{2731}{4096}}$$

18. Find the sum of the positive three digit odd integers.

101, 103, 105, ..., 999

$999=101+2(n-1)\Rightarrow n=450$

$$S_{450}=\frac{450(101+999)}{2}=247{,}500$$

19. On the first day of each year you invest \$1000 in an account that pays 6% interest compounded quarterly. How much is your account worth at the end of 20 years?

1.015

$$\underbrace{1000\left(1+\frac{0.06}{4}\right)^1}_{\text{LAST DEPOSIT}}+1000\left(1+\frac{0.06}{4}\right)^2+\cdots+\underbrace{1000\left(1+\frac{0.06}{4}\right)^{80}}_{\text{FIRST DEPOSIT}}$$

$$\therefore A=\frac{\overbrace{1000(1.015)}^{\text{FIRST TERM}}\left(1-1.015^{80}\right)}{1-1.015}$$

$$\approx\boxed{\$155001.52}$$

20. Evaluate $\sum_{n=1}^{\infty}\left(\frac{2}{5}\right)^n$

$|r|<1\ \therefore$ the sum exists

$$S_\infty=\frac{\left(\frac{2}{5}\right)}{1-\frac{2}{5}}=\frac{2}{5}\cdot\frac{5}{3}=\frac{2}{3}$$

$$\left(S_\infty=\frac{a_1}{1-r}\right)$$

21. Express $1.\overline{045}$ as a simplified fraction using the method of infinite series.

$1+0.045+0.00045+\cdots$

$r=0.001$

$$=1+\frac{0.045}{1-0.001}$$

$$=1+\frac{45}{999}$$

$$=1+\frac{5}{111}=\frac{116}{111}$$

22. Find the sum of the series $1000-500+250-\cdots+7.8125$ (FINITE)

Geometric $r=-\frac{1}{2}$

$7.8125=1000\left(-\frac{1}{2}\right)^{n-1}$

$0.0078125=\left(-\frac{1}{2}\right)^{n-1}$

$n-1$ must be <u>even</u> $\therefore$ drop the negative on r.

$0.0078125=\left(\frac{1}{2}\right)^{n-1}\Rightarrow n-1=\frac{\log 0.0078125}{\log\frac{1}{2}}$

$n=8$

$$S_8=\frac{1000\left(1-\left(-\frac{1}{2}\right)^8\right)}{1-\left(-\frac{1}{2}\right)}$$

$$=1000\cdot\frac{2}{3}\left(\frac{255}{256}\right)$$

$$=\boxed{\frac{10625}{16}}$$

23-24: A pile driver pounds a steel column into the earth. On its first drive, the column went down 1.5 m. On each successive drive the column moves 92% of its previous amount.

23. How far does it move on its 60th drive? Geo. w/ $r=0.92$

$a_{60}=1.5(0.92)^{60}=0.01\text{ m}$

(1 cm)

24. What total distance has the column traveled after 60 drives?

$$S_{60}=\frac{1.5\left(1-0.92^{60}\right)}{1-0.92}\approx\boxed{18.62\text{ m}}$$