Algebra 2 **TEST REVIEW Chapter 15A**

Name: SOLUTIONS **Section:** ________

SECTION 15-1 and 15-2: Statistics

The table below shows the final exam scores for a small Algebra 2 class.

Final Exam Score	100	66	96	65	58	70	80	78	96	41	80	66	99	55

1. Rewrite the data in order from least to greatest.

41, 55, 58, 65, 66, 66, 70 | 78, 80, 80, 96, 96, 99, 100

74

2. Make a stem and leaf plot of the data.

4	1
5	5 8
6	5 6 6
7	0 8
8	0 0
9	6 6 9
10	0

3. Determine the three measures of central tendency.

Mean $= \frac{1050}{14} = 75$

median $= \frac{70+78}{2} = 74$

mode $= 66, 80, 96$

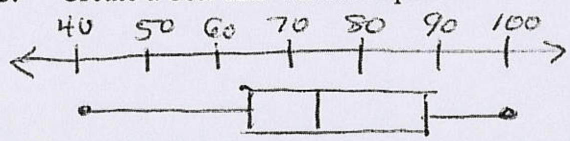
4. Find the first quartile, third quartile, and the inter-quartile range. (Include the median)

$Q1 = \frac{65+66}{2} = 65.5$

$Q3 = \frac{80+96}{2} = 88$

$IQR = 88 - 65.5 = 22.5$

5. Create a box-and-whiskers plot.



6. Determine the variance and standard deviation. Show all your calculations as we did in class.

DATA	DIFF FROM MEAN	DIFF SQUARED
41	41−75 = −34	1156
55	−20	400
58	−17	289
65	−10	100
66	−9	81
66	−9	81
70	−5	25
78	3	9
80	5	25
80	5	25
96	21	441
96	21	441
99	24	576
100	25	+ 625

n−1

$4274 \div 13 = \boxed{328.77}$ VARIANCE

$\sigma = \sqrt{328.77}$

$\boxed{\sigma = 18.13}$ STD. DEV

7. Suppose your test scores this semester are 78%, 83%, 92%, 65%, and 87%. What must you score on the sixth test to have an overall average of at least 83%?

$\frac{78+83+92+65+87+x}{6} = 83$

$405 + x = 498$

$\boxed{x = 93}$

8. Suppose 100 SPA students took a math test and the average was 80%. 50 students from a smaller school took the same test with an average of 70%. What is the average score for all students taken together?

mean $= \frac{100 \cdot 80 + 50 \cdot 70}{150}$

$= \boxed{76.\overline{6}}$

9. The data shows the number of times each score occurred on a test. Find the mean, median, and mode.

Score	10	15	20	25	30
Frequency	2	5	8	15	10
	20	75	160	375	300

Mean $= \frac{10 \cdot 2 + 15 \cdot 5 + 20 \cdot 8 + 25 \cdot 15 + 30 \cdot 10}{40} = 23.25$

median $= 25$ (where the 21^{st} data value is)

mode $= 25$ (occurs 15 times)

SECTION 15-5: Fundamental Counting Principles. You must show a complete setup to receive credit. You may use a calculator for any arithmetic.

10. How many odd numbers less than 10,000 can you make using only the digits 0, 3, 4, and 7?

Th H T O

$\boxed{3}\boxed{4}\boxed{4}\boxed{2} = 96$

$\boxed{3}\boxed{4}\boxed{2} = 24$

$\boxed{3}\boxed{2} = 6$

$\boxed{2} = 2$

$\boxed{128}$

11. In how many different ways can you answer a 12-question multiple choice test if each question has four choices and you can leave questions blank?

1 2 12

$\boxed{5}\boxed{5}\cdots\boxed{5}$

$5^{12} = \boxed{244{,}140{,}625}$

12. How many license plates having 3 symbols (letters and digits) can you make that have at least one digit?

mutually exclusive

1D $\overset{10\ 26\ 26}{DLL}$ LDL LLD $= 3 \cdot 6760$ +

2D $\overset{10\ 10\ 26}{DDL}$ DLD LDD $= 3 \cdot 2600$ +

3D $\overset{10\ 10\ 10}{DDD} = 1000$

$\boxed{29{,}080}$

13. How many three-letter "words" can you make if at least one of the letters must be a vowel? (do not count *y* as a vowel)

1V $\overset{5\cdot 21\cdot 21}{VCC}$ CVC CCV $= 3 \cdot 2205$ +

2V $\overset{5\ 5\ 21}{VVC}$ VCV CVV $= 3 \cdot 525$ +

3V $\overset{5\ 5\ 5}{VVV} = 125$

$\boxed{8315}$

SECTION 15-6: Permutations. Make sure to show a complete setup using "P" notation and all relevant factorials identified. You may use a calculator for any multiplication or division.

14. Evaluate $\dfrac{1000!}{995!}$ 9.9×10^{14}

$= \dfrac{1000 \cdot 999 \cdot 998 \cdot 997 \cdot 996 \cdot \cancel{995!}}{\cancel{995!}}$

15. Evaluate ${}_{100}P_2$. $= \dfrac{100!}{98!} = 100 \cdot 99$

$= 9900$

16. In how many ways can you arrange the letters in the word UNCOPYRIGHTABLE?

n = 15 none are identical

$\therefore {}_{15}P_{15} = 15!$

$= \boxed{1{,}307{,}674{,}368{,}000}$

17. How many seven letter words can you make from the letters in AMBIDEXTROUSLY?

n = 14 none Same

${}_{14}P_7 = \dfrac{14!}{7!} = \boxed{17{,}297{,}280}$

18. In how many ways can you arrange the letters in SESQUIPEDALIOPHOBIA? n = 19

$\dfrac{19!}{2!\,2!\,3!\,2!\,2!\,2!}$

S E I P A O

$= \boxed{6.336 \times 10^{14}}$

19. A box contains 8 identical red pens, 6 identical blue pens, and 10 other pens, all different. How many different ways are there of handing out those pens to a class of 24 students?

24 pens $\rightarrow \dfrac{24!}{\underbrace{8!\,6!}_{\text{Identicals}}}$

$= \boxed{2.137 \times 10^{16}}$

What is the meaning of sesquipedaliophobia? FEAR OF LONG WORDS

Algebra 2 **TEST REVIEW Chapter 15B**

Name: SOLUTIONS **Section:** ________

SECTION 15-7: Combinations

1. Evaluate ${}_{100}C_2$.

$= \frac{100!}{2!\,98!} = \frac{100 \cdot 99 \cdot \cancel{98!}}{2!\,\cancel{98!}}$

$= \boxed{4950}$

2. Find the number of three-letter combinations of the letters in the word STINGER. $n = 7$

${}_7C_3 = \frac{7!}{3!\,4!} = \frac{7 \cdot 6 \cdot 5 \cdot \cancel{4!}}{3 \cdot 2 \cdot 1\, \cancel{4!}} = 35$

3. You can order a hamburger with cheese, onion, pickles, ketchup, relish, mustard, lettuce, tomato, or mayonnaise. How many combinations of four "extras" can you make?

$n = 9$ extras

${}_9C_4 = \frac{9!}{4!\,5!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot \cancel{5!}}{4 \cdot 3 \cdot 2 \cdot 1\, \cancel{5!}}$

$= \boxed{126}$

4. 12 points lie randomly on the circumference of a circle. How many inscribed hexagons can you make using these points as vertices. (p. 740 in your text has a similar example). 6 sides

${}_{12}C_6 = \frac{12!}{6!\,6!} = \boxed{924}$

5. Suppose a club has 10 boys and 12 girls and needs to select a committee to decide on next year's activities. How many 6-person committees can you make if:

(A) All members are equally eligible?

${}_{22}C_6 = \frac{22!}{6!\,16!} = 74{,}613$

(B) There must be an equal number of boys and girls on the committee?

$\underbrace{\boxed{{}_{10}C_3}}_{\text{boys}} \cdot \underbrace{\boxed{{}_{12}C_3}}_{\text{girls}} = \frac{10!}{3!\,7!} \cdot \frac{12!}{3!\,9!}$

$= 120 \cdot 220$

$= \boxed{26{,}400}$

6. From a standard 52-card deck, how many 13-card hands can you make having exactly 8 clubs?

$\underbrace{\boxed{{}_{13}C_8}}_{\text{clubs}} \cdot \underbrace{\boxed{{}_{39}C_5}}_{\text{not clubs}} =$

$1287 \cdot 575{,}757 = \boxed{740{,}999{,}259}$

From the same deck, how many 5-card hands can you make with exactly 3 aces?

$\underbrace{\boxed{{}_4C_3}}_{\text{ACES}} \cdot \underbrace{\boxed{{}_{48}C_2}}_{\text{NOT ACES}}$

$= 4 \cdot 1128$

$= \boxed{4{,}512}$

7. Three dice are rolled, one green, one red, and the other blue. The outcome is an ordered triple (r, g, b).

(A) How many elements are there in the sample space? (**hint**: in how many ways can each die turn up?)

$\overset{r}{\boxed{6}} \cdot \overset{g}{\boxed{6}} \cdot \overset{b}{\boxed{6}} = 216$

(B) Write the event that all three dice show the same number.

$\{(1,1,1), (2,2,2), (3,3,3), (4,4,4), (5,5,5), (6,6,6)\}$

(C) Write the event that the red die is a multiple of 3, the green die is 6, and the blue die is even.

3 or 6 2, 4, 6

$\{(3,6,2), (3,6,4), (3,6,6), (6,6,2), (6,6,4), (6,6,6)\}$

8. A bag contains 4 red rocks, 5 blue rocks, 6 yellow rocks, and 7 green rocks. An experiment consists of drawing ONE rock and noting its color. Find the probability of each event.

A: The rock is not green. $\rightarrow$ R, B, Y

$P(\text{not green}) = \boxed{\frac{15}{22}}$

B: The rock is red or blue.

$P(R \text{ or } B) = \boxed{\frac{9}{22}}$

C: The rock is blue or not yellow. Red or green

$\frac{16}{22} = \frac{8}{11}$

D: The rock is a primary color. $\rightarrow$ R, Y, B

$\frac{4+5+6}{22} = \frac{15}{22}$

9. An experiment consists of tossing three coins one time and noting their outcomes.

Write the sample space for this experiment.

{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

(8 ways)

Find the probability of each event:

A: None are tails $\rightarrow$ all heads!

$\frac{1}{8}$

B: Exactly one is heads

THT, HTT 2 ways $\frac{2}{8} = \frac{1}{4}$

C: Exactly two are tails

HTT, THT, TTH $\frac{3}{8}$

D: At least two are tails

1T or 2T

$3 + 3 = \frac{6}{8} = \frac{3}{4}$

10. Using the same bag of rocks from above, a new experiment is done. This time, TWO rocks are drawn from the bag and each color is recorded.

How many elements are in the sample space?

${}_{22}C_2 = \frac{22 \cdot 21 \cdot \cancel{20!}}{2 \cdot \cancel{20!}} = 231$

Find the probability of each event:

A: Both are red

$\frac{{}_4C_2}{231} = \frac{6}{231} = \frac{3}{77}$

B: Neither is red

$\frac{{}_{18}C_2}{231} = \frac{153}{231} = \boxed{\frac{51}{77}}$

C: One is blue and one is yellow

$\frac{\boxed{{}_5C_1} \cdot \boxed{{}_6C_1}}{231} = \frac{30}{231} = \frac{10}{77}$

D: Both are blue or both are green mutually exclusive

$\frac{{}_5C_2 + {}_7C_2}{231} = \frac{10+21}{231} = \frac{31}{231}$

11. An experiment consists of randomly selecting 3 cards from a 52-card deck.

How many items are in the sample space?

${}_{52}C_3 = \frac{52 \cdot 51 \cdot 50}{6} = 22{,}100$

Find the probability of each event:

A: All three are hearts

$\frac{{}_{13}C_3}{22{,}100} = \frac{715}{22{,}100} = \boxed{\frac{143}{4420}} \approx 0.0324$

B: All three are kings

$\frac{{}_4C_3}{22{,}100} = \frac{4}{22{,}100} = \frac{1}{5{,}525}$

C: Two are diamonds and one is clubs

$\frac{\boxed{{}_{13}C_2} \cdot \boxed{13}}{22{,}100} = \frac{78 \cdot 13}{22{,}100}$

$= \frac{1014}{22{,}100} = \frac{507}{11{,}050}$

12. In a certain lottery game, a player wins the jackpot by matching the numbers on six balls chosen at random. The player chooses five numbers from 1 to 59 and a sixth number from 1 to 35. The player wins the jackpot by matching the first five numbers in any order and the sixth number. Find the probability of winning the jackpot. (Note: each number from 1 to 59 can only be used once)

$$\frac{59 \cdot 58 \cdot 57 \cdot 56 \cdot 55}{5!} \;\Big|\; 35$$

order does not matter so divide out the ways of ordering 5 items

OR $\boxed{{}_{59}C_5} \cdot \boxed{35}$

$$P(\text{Jackpot}) = \frac{1}{175{,}223{,}510}$$

13. In the game of poker, a full house is a five-card hand consisting of three cards of the same type and two cards of another type (i.e. three of a kind and a pair in the same hand). What is the probability of being dealt a full house from a fully shuffled 52-card deck? Express your answer as a decimal with three significant digits. "Bonus Level"

types for 3 of kind; types remaining for the pair

$$\frac{\boxed{13} \cdot \boxed{{}_4C_3} \cdot \boxed{12} \cdot \boxed{{}_4C_2}}{\boxed{{}_{52}C_5}}$$

$$\frac{13 \cdot 4 \cdot 12 \cdot 6}{2{,}598{,}960} = \frac{3744}{2{,}598{,}960} = \frac{6}{4165}$$

0.00144

SECTION 15-10: Mutually Exclusive and Independent Events

14. There are 3 red, 2 blue, and 3 yellow crayons in a box. Jeff randomly selects one, returns it to the box, and then randomly selects another. Find the probability of each event. 8 crayons

A: The first crayon selected is blue and the second is yellow.

Blue Yellow

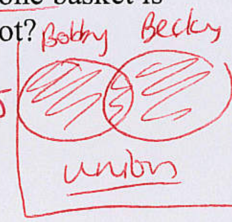
$$\frac{2}{8} \cdot \frac{3}{8} = \frac{6}{64} = \frac{3}{32}$$

B: Both crayons selected are red.

Red Red

$$\frac{3}{8} \cdot \frac{3}{8} = \frac{9}{64}$$

15. When Bobby shoots a basketball, the probability of a basket is 0.35. When Becky shoots, the probability of a basket is 0.55. What is the probability that at least one basket is made if they each take one shot?



Bobby misses $1 - 0.35 = .65$

Becky misses $1 - .55 = .45$

$1 - (.65)(.45)$

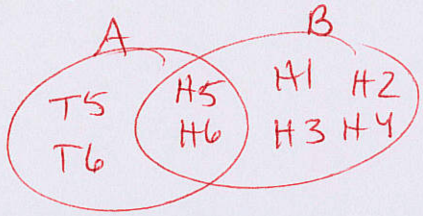
both miss

$1 - .2925 = .7075 \approx .71$

16. A die and a coin are tossed. Let A be the event that the die shows a 5 or 6, and let B be the event that the coin shows heads.

A: Specify the sample space for the experiment.

{T1, T2, T3, T4, T5, T6,
H1, H2, H3, H4, H5, H6}



B: Specify the simple events in A, B, $A \cup B$, and $A \cap B$.

A {T5, T6}

B {H1, H2, H3, H4, H5, H6}

$A \cup B$ {H1, H2, H3, H4, H5, H6, T5, T6}

$A \cap B$ {H5, H6}

C: Find the probability of A, B, $A \cup B$, and $A \cap B$.

$P(A) = \frac{2}{12} = \frac{1}{6}$

$P(B) = \frac{6}{12} = \frac{1}{2}$

$P(A \cup B) = \frac{8}{12} = \frac{2}{3}$

$P(A \cap B) = \frac{2}{12} = \frac{1}{6}$

D: Are A and B mutually exclusive? Are they independent?

They are NOT mutually exclusive because there are items in the intersection $A \cap B$.

Independent: IFF $P(A \cap B) = P(A) \cdot P(B)$

NOT independent

$\frac{1}{6} \stackrel{?}{=} \frac{1}{6} \cdot \frac{1}{2}$

$\frac{1}{6} = \frac{1}{12}$

17. The probability that Karla will ask Frank to be her tennis partner is ¼, that Juan will ask Frank is ⅓, and the Roger will ask Frank is ¾. Find the probability of each event.

KF = 1/4
JF = 1/3
RF = 3/4

A: Karla and Juan ask Frank.

$\frac{1}{4} \cdot \frac{1}{3} = \frac{1}{12}$

B: Juan and Roger ask Frank, but Karla doesn't.

$\frac{1}{3} \cdot \frac{3}{4} \cdot \left(1 - \frac{1}{4}\right)$

$\cancel{\frac{1}{3}} \cancel{\frac{3}{4}} \cdot \frac{3}{4} = \frac{3}{16}$

C: At least two of the three ask Frank. 19/48

$\underbrace{KJ}_{\text{not } R}$ or $\underbrace{KR}_{\text{not } J}$ or $\underbrace{JR}_{\text{not } K}$ or $\underbrace{KJR}_{\text{all}}$

$\left(1-\frac{3}{4}\right)\left(\frac{1}{3}\right)\left(\frac{1}{4}\right) + \left(1-\frac{1}{3}\right)\left(\frac{1}{4}\right)\left(\frac{3}{4}\right) + \left(1-\frac{1}{4}\right)\left(\frac{1}{3}\right)\left(\frac{3}{4}\right) + \left(\frac{1}{4}\right)\left(\frac{1}{3}\right)\left(\frac{3}{4}\right) = \frac{1}{48} + \frac{6}{48} + \frac{9}{48} + \frac{3}{48} = \frac{19}{48}$

D: At least one of the three asks Frank.

not K $\rightarrow 1 - \frac{1}{4} = \frac{3}{4}$

not J $\rightarrow 1 - \frac{1}{3} = \frac{2}{3}$

not R $\rightarrow 1 - \frac{3}{4} = \frac{1}{4}$

nobody $\rightarrow \frac{3}{4} \cdot \frac{2}{3} \cdot \frac{1}{4} = \frac{2}{16} = \frac{1}{8}$

$1 - \frac{1}{8} = \frac{7}{8} \approx .88$